Resolving the HANK Trilemma: The Role of Hours Complementarities *

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Abstract

Sticky-price heterogeneous-agent New Keynesian (HANK) models struggle to simultaneously match the empirical magnitudes of the average consumption response to unanticipated wealth shocks (MPC), the average short-run labor earnings response to these shocks (MPE), and the aggregate output response to government spending shocks. This tension is known as the Auclert–Bardóczy–Rognlie HANK trilemma. The canonical sticky-wage solution assumes uniform hours, which arise when workers' hours are perfect complements. However, existing empirical evidence shows that hours vary across workers and are gross but imperfect complements. In this paper, I introduce imperfect complementarity between hours into otherwise standard sticky-price and sticky-wage HANK models. Hours complementarities in the range of micro estimates close half to three-quarters of the gap between the baseline MPE and the data. They do so without inflating fiscal multipliers because they reduce the wealth effect of idiosyncratic but not aggregate shocks.

Keywords: HANK trilemma, hours complementarities, labor supply elasticity, marginal propensity to earn, fiscal multipliers
JEL codes: E24, E62, H31, H32

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1. Introduction

Auclert et al. (2023) show that standard sticky-price, flexible-wage heterogeneous-agent New Keynesian (HANK) models face a trilemma: they struggle to simultaneously match (i) a quarterly marginal propensity to consume (MPC) of 0.25, (ii) an annual marginal propensity to earn (MPE) of 0.023, and (iii) present-value fiscal multipliers no higher than 1.5.[1] Both MPCs and MPEs are critical moments for standard HANK models to match because they directly shape the general-equilibrium output responses to government spending shocks. Matching empirical evidence on fiscal multipliers is equally important for a credible analysis of fiscal policy. The trilemma arises because, at micro estimates of the Frisch elasticity, matching both the MPC and the MPE requires nonseparable preferences that reduce the wealth effect on labor supply and thereby inflate fiscal multipliers, contradicting fact (iii).

The canonical sticky-wage solution of Auclert et al. (2023) sidesteps the trilemma by assuming workers' hours are uniform. Workers are then off their labor supply curves, and the MPE is zero by construction.[2] In this paper, I show that uniform hours arise when workers' hours are perfect complements, such as when workers must coordinate their schedules. Yet, hours vary across workers in the data, and existing empirical evidence shows that workers' hours are gross but imperfect complements in production (Shao et al., 2023; Battisti et al., 2024). Motivated by this evidence, I ask whether empirically grounded complementarities between workers' hours can solve the HANK trilemma.

To address this question, I incorporate hours complementarities into the labor aggregation in both sticky-price and sticky-wage HANK models. Considering both frameworks allows me to disentangle the effects of hours complementarities and sticky wages. I begin with a standard sticky-price framework that combines the incomplete-markets household block with monopolistically competitive firms and sticky prices as in Rotemberg (1982). I depart from the standard assumption of linear labor aggregation and let firms aggregate labor using a CES technology that makes

[1] The MPC is the average consumption response to unanticipated wealth shocks. Kaplan and Violante (2022) review the existing empirical evidence about MPCs in the context of heterogeneous-agent models and conclude that the MPC on nondurable goods and services is between 15% and 25%. The MPE is the average short-run labor earnings response to these shocks, defined as the share of a wealth shock spent on reducing labor income. The evidence comes from studies of lottery winners (Imbens et al., 2001; Cesarini et al., 2017; Picchio et al., 2018; Golosov et al., 2024). The value of 0.023 is the US estimate from Golosov et al. (2024). The present-value fiscal multiplier is the output response to government spending shocks, typically estimated below one and no higher than 1.5 at the zero lower bound (Ramey and Zubairy, 2018; Ramey, 2019).

[2] The model matches facts (i) and (iii) by assuming separable preferences.

workers more productive when they work similar hours. This departure is consistent with empirical evidence in Shao et al. (2023) and Kuhn et al. (2023) that working hours are complements in production. The sticky-wage model with heterogeneous hours has the same block structure as the sticky-price model, but unions rather than firms aggregate labor using the CES technology.

I calibrate both models to a quarterly MPC of 0.25, matching fact (i), along with US taxes, transfers, liquid asset supply, and labor earnings dispersion. I use the same calibration targets as Auclert et al. (2023) to make my results directly comparable with their sticky-price and sticky-wage models. To understand how much complementarity is required to solve the HANK trilemma, I compute the MPEs and fiscal multipliers across a range of values for the elasticity of substitution between hours, including estimates from the empirical literature. I assume that monetary policy follows a constant real rate rule.

In the sticky-price model with separable preferences, empirically grounded hours complementarities close half to three-quarters of the gap between the baseline MPE and the data, while still matching facts (i) and (iii). The MPE in the baseline model is 0.216, compared with 0.023 in the data (Golosov et al., 2024). Calibrating the elasticity of substitution between hours worked to the estimates in Shao et al. (2023) and Battisti et al. (2024) yields MPEs of 0.113 and 0.075, respectively. However, matching the MPE in the data requires an elasticity of substitution below 0.125, substantially lower than existing empirical estimates. Fiscal multipliers remain between 1.24 and 1.28, consistent with fact (iii).

Empirically grounded hours complementarities reduce the fiscal multiplier in the sticky-price model with a highly accommodative monetary policy by more than an order of magnitude. I consider a three-year nominal interest rate peg. The central bank holds the nominal interest rate fixed for three years and follows a Taylor rule thereafter. This policy is more accommodative than the constant real rate rule, and the standard sticky-price model with separable preferences generates a present-value government spending multiplier of 73.92. With hours complementarities consistent with the data, it falls between 2.40 and 2.53, closer to but still above the empirical upper bound of 1.5.

With separable preferences, sticky wages alone cannot solve the HANK trilemma, while hours complementarities generate MPEs almost identical to those in the sticky-price model, alleviating the trilemma. I calibrate the sticky-wage model with heterogeneous hours and find an MPE of 0.241. Empirically grounded complementarities lower the MPE to between 0.077 and 0.116. At the same time, present-value fiscal multipliers stay between 1.00 and 1.04 under the

constant real rate rule and between 1.14 and 1.25 under the three-year nominal interest rate peg, consistent with fact (iii).

A discussion of the mechanism. Hours complementarities alleviate the HANK trilemma by making individual hours respond less to idiosyncratic shocks, such as winning a lottery, than aggregate hours do to aggregate shocks. In Section 2, I build an analytical framework to understand when wage-hours dependence can help solve the trilemma. Wage-hours dependence means that the hourly wage rate depends on own and other workers' hours. Hours complementarities are one mechanism that can generate it. Since workers are more productive when they work similar hours, individual incentives to work decrease with own hours but increase with other workers' hours. Decreasing returns to own hours reduce the labor supply elasticity to idiosyncratic wage shocks and lower the MPE-MPC ratio, which increases with that elasticity. At the same time, hours complementarities introduce a positive feedback effect from aggregate to individual hours. With CES hours aggregation, the feedback effect exactly offsets the decreasing returns, so hours complementarities do not move the elasticity of aggregate hours to aggregate wage shocks. The improvement in the MPE therefore does not come at a cost of matching how strongly aggregate hours respond to transitory aggregate wage shocks.

Hours complementarities lower the MPE without inflating fiscal multipliers. Government spending affects aggregate hours through the wage and wealth effects. In HANK models with linear labor aggregation, government spending increases wages, which induces wealth and wage effects on labor supply of opposite sign. Nonseparable preferences reduce the responsiveness of hours to changes in wealth but not to wages, and the fiscal multiplier explodes. Hours complementarities, however, scale the responses of individual hours to changes in wages and wealth in equal proportion. They also do not affect the wage elasticity of aggregate hours, which scales both effects. Therefore, fiscal multipliers remain consistent with fact (iii).

Related literature. This paper contributes to three strands of the literature. The first strand establishes the HANK trilemma and studies labor supply in sticky-wage HANK models. Auclert et al. (2023) identify the HANK trilemma for sticky-price New Keynesian models and propose a sticky-wage resolution that restricts labor supply by assuming unions impose uniform hours. Bilbiie et al. (2025) argue that the trilemma can be resolved by preferences that parameterize the wealth effect on labor supply separately from complementarity between consumption and hours, which allows the model to match both the MPE and the fiscal multiplier. Gerke et al. (2024) relax the uniform hours assumption and show that heterogeneous hours in a sticky-wage

model attenuate aggregate responses to monetary policy shocks, but do not report the model's fit to the MPE. Faia and Shabalina (2025) show that a sticky-wage model with heterogeneous hours does not match the MPE in the data and argue that a model with uniform hours and a labor force participation margin can solve the trilemma. Relatedly, Graves et al. (2023) and Alves and Violante (2025) develop models with labor market frictions and an extensive margin of labor supply in which the MPE is well below the MPC, breaking the tight link between the two. A common feature of both frameworks is that households cannot individually adjust their hours worked. I contribute to this literature by showing that hours complementarities in the labor aggregation, by firms in the sticky-price model and by unions in the sticky-wage model, alleviate the HANK trilemma, while allowing hours to vary across households. I also show that perfect complementarity between workers' hours in the unions' labor aggregation microfounds the uniform hours assumption that plays an important role in the canonical sticky-wage model.

The second strand studies complementarities between workers' hours in production. Yurdagul (2017) formalizes hours complementarities through a productivity-weighted CES aggregator over individual hours and uses it to explain why entrepreneurs in the US earn less on average than salaried workers. Bick et al. (2022) and Yurdagul (2017) show that in the cross-section of US workers, hourly wages are hump-shaped in hours worked, consistent with hours complementarities.[3] Shao et al. (2023) and Battisti et al. (2024) provide direct empirical evidence for the mechanism using matched employer-employee data from Canada and Italy, respectively. They find that workers' hours are gross complements in firms' production. The key mechanism in my paper, the distinction between the idiosyncratic and aggregate Frisch elasticities, draws on the contrast between idiosyncratic and firm-level variation in hours emphasized by Battisti et al. (2024), whose estimates I use to calibrate my model. I contribute to this literature by studying how hours complementarities affect labor supply and fiscal multipliers in HANK models.

The third strand of the literature incorporates nonlinear mappings from hours to wages in business-cycle models, and dates back to the seminal work of Kydland and Prescott (1991). In the heterogeneous-agent literature, recent work examines how nonlinear earnings, which activate the intensive and extensive margins of labor supply, affect wealth inequality and the welfare costs of business cycles (Ma, 2026) as well as monetary policy transmission (Ma, 2025). I contribute to this literature by microfounding the nonlinear mapping from hours to labor earnings through

[3] Cubas et al. (2021) and Cubas et al. (2023) provide related evidence on the timing of hours: mothers who miss peak hours of work earn lower hourly wages, which contributes to the gender wage gap.

hours complementarities, and studying how it affects the MPE–fiscal multiplier trade-off.

Road map. The paper first shows analytically that wage-hours dependence can help solve the HANK trilemma (Section 2), then presents the quantitative HANK model with hours complementarities that microfound wage-hours dependence (Section 3), evaluates its implications for Frisch elasticities, MPEs, and fiscal multipliers (Sections 4 and 5), and finally relates them to the canonical sticky-wage solution (Section 6).[4]

2. Wage-Hours Dependence and the HANK Trilemma

In this section, I show analytically that wage-hours dependence can help solve the HANK trilemma. Wage-hours dependence means that the hourly wage rate depends on individual as well as other workers' hours. Hours complementarities are one mechanism that can generate it. I outline the argument in three steps. First, I show that whenever individual incentives to work increase with aggregate hours, a positive feedback effect makes the elasticity of aggregate hours to wage shocks common to all households higher than the elasticity of individual hours to idiosyncratic wage shocks. Second, I show that the MPE-MPC ratio depends on the elasticity of individual hours to idiosyncratic wage shocks. Third, I show that the fiscal multiplier depends on the elasticity of aggregate hours to wage shocks common to all households. The aggregate elasticity scales both the wealth and wage effects of government spending. Therefore, unlike nonseparable preferences that only reduce the wealth effect, wage-hours dependence can in principle lower the MPEs without inflating fiscal multipliers.

2.1. Households

Consider a unit mass of infinitely lived, single-member households. They choose consumption c, hours h, and next-period assets a' to maximize expected discounted utility subject to budget and borrowing constraints, where $U(c, h)$ is a per-period utility function. Individual state variables are assets a and idiosyncratic productivity x. Households take the aggregate wage index $\tilde{w}$, the flat income tax rate τ^w, and aggregate hours H as given. Let the after-tax wage index $\tilde{w}^\tau \equiv (1 - \tau^w)\tilde{w}$ denote the product of the aggregate wage index and the tax factor. Let the

[4]Besides complementarity between workers' hours, several mechanisms can make hourly wages depend on individual and aggregate hours. Work set-up costs, minimum-hours requirements, and fatigue can make hourly wages depend on individual hours. Likewise, standard workweeks induced by collective bargaining or work-related norms can make hourly wages depend on the distribution of hours.

after-tax wage factor $\tilde{w}^{\tau,x} \equiv \tilde{w}^{\tau} x$ denote its product with idiosyncratic productivity. Each period, households earn after-tax labor income $y(h, \tilde{w}^{\tau,x}, H)$, asset income ra, and receive transfers T.

After-tax labor earnings equal the product of the after-tax hourly wage rate $(1-\tau^{w})w(h, x, H)$ and hours worked h. The pre-tax hourly wage rate $w(h, x, H)$ is a product of the aggregate wage index $\tilde{w}$, idiosyncratic productivity x, and the wage-hours dependence function $f(h, H)$.

$$y(h, \tilde{w}^{\tau,x}, H) \equiv (1 - \tau^{w})w(h, x, H)h, \tag{1}$$

$$w(h, x, H) \equiv \tilde{w} x f(h, H), \tag{2}$$

Workers therefore face a wage schedule rather than a single wage.

Aggregate hours H are a sufficient statistic because they fully describe how changes in the hours distribution affect household labor earnings. I assume that aggregate hours are homogeneous of degree one, so they move as a weighted average of individual responses.[5] I also assume that the first-order conditions are necessary and sufficient.[6]

The household problem has the following recursive representation

$$V(a, x) = \max_{c,h,a'} U(c, h) + \beta \mathbb{E}\left[V(a', x') \mid x\right] \tag{3}$$

$$\text{s.t.} \quad c + a' = (1 + r)a + \underbrace{(1 - \tau^{w})w(h, x, H)h}_{y(h,\tilde{w}^{\tau,x},H)} + T$$

$$a' \geq \underline{a}.$$

The marginal utility of consumption equals the Lagrange multiplier on the budget constraint λ:

$$U_c(c, h) = \lambda. \tag{4}$$

Households choose hours worked to equalize the marginal disutility of working extra time and the marginal gain in utility from consumption associated with higher earnings:

$$-U_h(c, h) = \lambda \underbrace{\tilde{w}^{\tau,x}\left(\overbrace{f(h, H)}^{\text{direct}} + \overbrace{f_h(h, H)\, h}^{\text{indirect}}\right)}_{y_h}. \tag{5}$$

[5]The CES aggregator I use in the quantitative model of Section 3 satisfies homogeneity of degree one, and the standard linear labor aggregator is a special case.

[6]The utility function U is strictly concave, and the labor earnings function is concave, with strictly positive marginal earnings. At zero hours, the marginal disutility of working is zero. An interior solution for hours means I restrict attention to the intensive margin of labor supply, as in the quantitative model of Section 3. Appendix A.1 states the assumptions formally.

In the standard model, the hourly wage rate is independent of hours, and marginal earnings equal the after-tax hourly wage rate. With wage-hours dependence, by contrast, marginal earnings also depend on how hours affect hourly wages.

Conditions (4) and (5) implicitly define the Frisch consumption-demand $c(\lambda, \tilde{w}^{\tau,x}, H)$ and hours-supply $h(\lambda, \tilde{w}^{\tau,x}, H)$ functions. I use them to define Frisch elasticities, which hold the marginal utility of consumption λ fixed. I will first study the Frisch elasticities of labor supply.

2.2. Idiosyncratic and Aggregate Frisch Elasticities

In this section, I define idiosyncratic and aggregate Frisch elasticities and show that the former are lower than the latter when individual incentives to work increase with aggregate hours. Let the *idiosyncratic Frisch elasticity*,

$$\text{Frisch}_i \equiv \frac{\partial \log h(\lambda, \tilde{w}^{\tau,x}, H)}{\partial \log \tilde{w}^{\tau,x}}, \tag{6}$$

measure how labor supply responds to idiosyncratic transitory wage shocks that do not affect the marginal utility of consumption λ and aggregate hours H.[7] And let the *aggregate Frisch elasticity*,

$$\text{Frisch}_a \equiv \left.\frac{d \log H}{d \log \tilde{w}^{\tau}}\right|_{\{\lambda\}=\{\bar{\lambda}\}}, \tag{7}$$

measure how aggregate hours respond to transitory wage shocks common to all households, holding all households' marginal utility of consumption fixed.

First, when labor earnings display decreasing returns to hours, $y_{hh} < 0$, the idiosyncratic Frisch elasticity is lower than in the standard model,

$$\text{Frisch}_i = \left(\frac{U_{hh} h}{U_h} - \frac{U_{ch}^2 h}{U_{cc} U_h} - \frac{y_{hh} h}{y_h}\right)^{-1}.$$

The intuition is that the earnings gain from working longer decreases with hours, making the optimal hours adjustment smaller than in the standard model.[8]

Second, when marginal earnings y_h increase with aggregate hours H, *aggregate* Frisch elasticity is higher than *idiosyncratic* Frisch elasticity. The after-tax wage factor $\tilde{w}^{\tau,x}$, aggregate

[7] Since the tax factor, idiosyncratic productivity x, and the aggregate wage index $\tilde{w}$ only enter the problem through the after-tax wage factor $\tilde{w}^{\tau,x}$ in the budget constraint, the idiosyncratic Frisch elasticities to each product component coincide.

[8] I prove the result in Appendix A.2.

hours H, and the marginal utility of consumption λ affect individual hours $h(\lambda, \tilde{w}^{\tau,x}, H)$ in proportion to the idiosyncratic Frisch elasticity,

$$\begin{aligned} d\log h(a,x) = &\text{Frisch}_i(a,x)\left(d\log\tilde{w}^{\tau,x} + \epsilon_{y_h,H}(a,x)\, d\log H\right) \\ &+ (1-\text{CI}(a,x))\,\text{Frisch}_i(a,x)\, d\log\lambda(a,x), \end{aligned} \tag{8}$$

where the *complementarity index* CI,

$$\text{CI} \equiv \frac{\partial c(\lambda, \tilde{w}^{\tau,x}, H)}{\partial \tilde{w}^{\tau,x}} \Bigg/ y_h \frac{\partial h(\lambda, \tilde{w}^{\tau,x}, H)}{\partial \tilde{w}^{\tau,x}}, \tag{9}$$

measures the marginal propensity to consume out of compensated labor income in response to changes in the after-tax wage factor $\tilde{w}^{\tau,x}$ and regulates the strength of the wealth effect on labor supply.[9] The elasticity of marginal earnings to aggregate hours,

$$\epsilon_{y_h,H}(a,x) \equiv \left.\frac{\partial y_h}{\partial H}\right|_{h=\bar{h},\, \tilde{w}^{\tau,x}=\bar{\tilde{w}}^{\tau,x}} \frac{H}{y_h}, \tag{10}$$

scales the effect of aggregate hours on individual hours. To first order, log changes in aggregate hours equal the weighted average of log changes in individual hours,

$$\begin{aligned} d\log H &= \int\int d\log h(a,x)\,\omega(a,x)\,da\,dx, \\ \text{with}\quad \omega(a,x) &\equiv \frac{\partial H}{\partial h(a,x)}\frac{h(a,x)}{H}, \end{aligned} \tag{11}$$

where the weights $\omega(a,x)$ integrate to one because the aggregator is homogeneous of degree one.[10] Conditions (8) and (11) jointly determine the movement in aggregate hours.

Proposition 1. *Let* $\overline{\text{Frisch}_i}$ *denote the weighted average idiosyncratic Frisch elasticity and let* Γ *denote the weighted average household labor supply elasticity to aggregate hours,*

$$\overline{\text{Frisch}_i} \equiv \int\int \text{Frisch}_i(a,x)\,\omega(a,x)\,da\,dx, \tag{12}$$

$$\Gamma \equiv \int\int \text{Frisch}_i(a,x)\,\epsilon_{y_h,H}(a,x)\,\omega(a,x)\,da\,dx, \tag{13}$$

with weights $\omega(a,x)$ *given by* (11)*. If* $|\Gamma| < 1$*, the aggregate Frisch elasticity is finite and equals*

$$\text{Frisch}_a = \frac{\overline{\text{Frisch}_i}}{1-\Gamma}, \tag{14}$$

[9] The complementarity index depends on the cross-partial derivative of the utility function with respect to consumption and hours: $\text{CI} = U_{ch}U_c/U_{cc}/U_h$. Appendix A.6 proves the result.

[10] I prove the result in Appendix A.3.

and the hours of household (a, x) *respond to the same shock according to*

$$\left.\frac{d \log h(a,x)}{d \log \tilde{w}^{\tau}}\right|_{\{\lambda\}=\bar{\lambda}} = \text{Frisch}_i(a,x)\left(1 + \epsilon_{y_h,H}(a,x)\,\text{Frisch}_a\right). \tag{15}$$

If in addition $\epsilon_{y_h,H}$ *is common across households, then* $\Gamma = \overline{\text{Frisch}_i}\,\epsilon_{y_h,H}$ *and*

$$\frac{1}{\overline{\text{Frisch}_i}} - \frac{1}{\text{Frisch}_a} = \epsilon_{y_h,H}. \tag{16}$$

Proposition 1 states that the aggregate Frisch elasticity is the average idiosyncratic Frisch elasticity amplified by the feedback between aggregate and individual hours.[11] If aggregate hours do not affect marginal earnings, $\epsilon_{y_h,H} = 0$, there is no feedback effect, and aggregate Frisch elasticity equals the average idiosyncratic Frisch elasticity. However, if marginal earnings increase with aggregate hours, $\epsilon_{y_h,H} > 0$, the positive feedback effect amplifies the aggregate elasticity. The mechanism is a standard multiplier effect. A common one percent rise in $\tilde{w}^{\tau}$ raises each household's log hours by its own idiosyncratic Frisch elasticity and aggregate hours by the average idiosyncratic Frisch elasticity $\overline{\text{Frisch}_i}$. A $\overline{\text{Frisch}_i}$ increase in aggregate hours raises marginal earnings, inducing a further $\Gamma\,\overline{\text{Frisch}_i}$ percent increase in aggregate hours, and so on.[12]

2.3. The MPE-MPC Ratio

Static optimality conditions are sufficient to characterize the ratio of households' instantaneous labor earnings and consumption responses to unanticipated wealth shocks.[13] I define the marginal propensity to earn (MPE) as a measure of how much after-tax labor earnings decline in response to a wealth shock ∂T:

$$\text{MPE} \equiv -y_h \frac{\partial h(a,x,T)}{\partial T}.$$

I define the marginal propensity to consume (MPC) as the responsiveness of household consumption to a wealth shock:

$$\text{MPC} \equiv \frac{\partial c(a,x,T)}{\partial T}.$$

The elasticity of intertemporal substitution,

$$\text{EIS} \equiv -\frac{\partial \log c(\lambda, \tilde{w}^{\tau,x}, H)}{\partial \log \lambda},$$

[11] I prove Proposition 1 in Appendix A.5.

[12] Summing the sequence yields a standard multiplier effect, $\text{Frisch}_a = \overline{\text{Frisch}_i} \sum_{k=0}^{\infty} \Gamma^k = \overline{\text{Frisch}_i}/(1-\Gamma)$, with the geometric sum converging if the feedback effect $|\Gamma|$ is below one.

[13] The state-of-the-art evidence about MPCs and MPEs comes from studies of lottery winners, which are unlikely to affect prices.

is the negative of the elasticity of consumption with respect to the Lagrange multiplier on the budget constraint λ.

Proposition 2. *For any individual in the state* (a, x)*:*

$$\frac{\text{MPE}}{\text{MPC}} = \frac{y(h, \tilde{w}^{\tau,x}, H)\left(1 + \epsilon_{w,h}\right)}{c} \frac{\text{Frisch}_i}{\text{EIS}} (1 - \text{CI}) \tag{17}$$

The idiosyncratic Frisch elasticity determines the MPE-MPC ratio because the wealth variation is idiosyncratic and does not affect aggregate hours H. Therefore, the more marginal earnings y_h decrease with hours, the lower the idiosyncratic elasticity, the lower the MPE-MPC ratio.[14]

2.4. Aggregate Frisch and Fiscal Multipliers

While the MPE-MPC ratio depends on the idiosyncratic Frisch elasticity, fiscal multipliers depend on the aggregate Frisch elasticity. Integrating equation (8) over all households, varying the common wage component so that $d \log \tilde{w}^{\tau,x} = d \log \tilde{w}^{\tau}$, and assuming that the idiosyncratic Frisch elasticity is constant across agents yields the following expression for a change in log aggregate hours:

$$\begin{aligned} d \log H = &\text{Frisch}_a \, d \log \tilde{w}^{\tau} \\ &+ \text{Frisch}_a \int \int (1 - \text{CI}(a, x)) \, d \log \lambda(a, x) \, \omega(a, x) \, da \, dx, \end{aligned} \tag{18}$$

which depends only on the aggregate Frisch elasticity.[15]

Proposition 3. *Let the idiosyncratic Frisch elasticity be constant across households, and let output be linear in the labor input up to a fixed cost,* $Y = zH - F$.[16] *Then, the impact multiplier of government spending can be decomposed into the general-equilibrium wage and wealth effects:*

$$\frac{dY}{dG} = \text{Frisch}_a \frac{zH}{G} \left[\Omega_{\tilde{w}^{\tau},G} + \int \int (1 - \text{CI}(a, x)) \, \Omega_{\lambda,G}(a, x) \, \omega(a, x) \, da \, dx \right], \tag{19}$$

[14] The MPE-MPC ratio also increases with the elasticity of the hourly wage rate to hours worked, $\epsilon_{w,h} \equiv \partial \log w(h, x, H) / \partial \log h$, and decreases with the complementarity index and the elasticity of intertemporal substitution. I prove the result and discuss each term in Appendix A.7 and Appendix A.8, respectively.

[15] If the idiosyncratic Frisch elasticity differs between individuals, the response of aggregate hours additionally depends on how the complementarity index covaries with the idiosyncratic Frisch elasticity. I derive the general formula in Appendix A.5.

[16] In the quantitative model, Frisch_i is uniform across households, and the aggregate production function is $Y_t = zH_t - F$.

where $\Omega_{\tilde{w}^\tau,G} \equiv d\log\tilde{w}^\tau/d\log G$ *and* $\Omega_{\lambda,G}(a,x) \equiv d\log\lambda(a,x)/d\log G$ *denote the general-equilibrium effects of government spending on the after-tax wage index and the marginal utility of consumption.*[17]

Proposition 3 indicates that wage-hours dependence does not directly increase the impact multiplier. Thus, the wedge between the idiosyncratic and aggregate Frisch elasticities opens up the possibility of generating the MPC, MPE, and fiscal multipliers consistent with the data without reducing the aggregate Frisch elasticity. Proposition 1 shows that whenever individual incentives to work increase with aggregate hours, the aggregate elasticity exceeds the idiosyncratic elasticity. Proposition 2 shows that the MPE-MPC ratio depends on the idiosyncratic Frisch elasticity, and Proposition 3 shows that the fiscal multiplier depends on the aggregate Frisch elasticity, which scales both the wage and wealth effects of government spending on aggregate hours.[18] Although wage-hours dependence offers a possible resolution of the HANK trilemma, the fiscal multiplier ultimately depends on the general-equilibrium feedback effects $\Omega_{\tilde{w}^\tau,G}$ and $\Omega_{\lambda,G}$, which themselves depend on preferences and technology. Hence, answering whether wage-hours dependence resolves the HANK trilemma requires specifying a full quantitative model, which I do in Section 3.

3. Sticky-Price HANK Model with Hours Complementarities in Production

In this section, I build a one-asset sticky-price HANK model that incorporates complementarity between workers' hours in production.[19] This mechanism is well supported by existing empirical literature. Bick et al. (2022) show that hourly wage rates are hump-shaped in hours worked.[20] They also observe that wage-hours profiles are systematically related to standard workweeks, suggesting a potential relationship between aggregate hours and marginal earnings. Moreover, a growing empirical literature tests the mechanism directly in the data and finds that hours are gross complements in production (Shao et al., 2023; Battisti et al., 2024).

The objective is to evaluate quantitatively whether hours complementarities can solve the HANK trilemma of Auclert et al. (2023). Hours complementarities enter the model in the

[17] I prove the result in Appendix A.9. See Azinovic-Yang et al. (2026) for a similar decomposition.

[18] To the extent that measured Frisch elasticities capture wage variation that affects aggregate hours H, equation (15) shows that they can exceed the idiosyncratic Frisch elasticity that regulates the MPE. I revisit this point in greater detail when discussing the mechanics of the quantitative model in Section 5.3.

[19] I present and discuss a sticky-wage version of the model in Section 6.

[20] The result is robust to accounting for the self-selection of more productive workers into longer hours.

production of the intermediate goods. I retain the model structure from the sticky-price one-asset HANK model of Auclert et al. (2023), which they used to establish the HANK trilemma. Since I assume certainty equivalence at the aggregate level, aggregate risk does not enter the households' and firms' problems.

3.1. Firms

There are two types of firms: final-goods and intermediate-goods producers.

Final-goods producers. Final-goods producers combine intermediate varieties indexed by $j \in [0, 1]$ into a single aggregate consumption good using a CES technology,

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}},$$

where $\varepsilon_p > 1$ denotes the elasticity of substitution between varieties. Competitive final-goods producers choose the quantity of each variety y_{jt} to maximize static profits,

$$\max_{\{y_{jt}\}_{j \in [0,1]}} P_t \left(\int_0^1 y_{jt}^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} - \int_0^1 p_{jt} y_{jt} dj,$$

and the optimal demand schedule for variety j is given by

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\varepsilon_p} Y_t.$$

Zero profits in the competitive final-goods sector yield the standard aggregate price index,

$$P_t = \left(\int_0^1 p_{jt}^{1-\varepsilon_p} dj \right)^{\frac{1}{1-\varepsilon_p}}.$$

Intermediate-goods producers. Intermediate-goods producers are monopolistically competitive and face the CES demand from final-goods producers. The production function for firm j, producing a single intermediate good, is linear in labor input n_{jt} with aggregate productivity z and a fixed production cost F,

$$y_{jt} = z n_{jt} - F.$$

Following Yurdagul (2017) and Shao et al. (2023), the labor input incorporates potential production-side complementarities across individual hours worked through a productivity-weighted CES aggregator over worker-level hours,

$$n_{jt} = \left(\int_{x \in \mathcal{X}} \int_{h \in \mathcal{H}} x \mu^j(h, x) h^{\frac{\sigma_h - 1}{\sigma_h}} dh dx \right)^{\frac{\sigma_h}{\sigma_h - 1}} \left(\int_{x \in \mathcal{X}} \int_{h \in \mathcal{H}} x \mu^j(h, x) dh dx \right)^{-\frac{1}{\sigma_h - 1}},$$

where $\mu^j(h,x)$ is the measure of workers supplying h hours with idiosyncratic productivity x, and σ_h denotes the elasticity of substitution between individual hours worked.[21] This formulation captures the idea that time spent at work is more productive when co-workers are simultaneously present. The labor input can be equivalently decomposed into total headcount, average idiosyncratic productivity, and effective hours per worker:

$$n_{jt} = \underbrace{\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} d\mu^j(h,x)}_{\text{Headcount}} \cdot \underbrace{\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} \frac{x\,d\mu^j(h,x)}{\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} d\mu^j(h,x)}}_{\text{Average idiosyncratic productivity}} \cdot \underbrace{H_{jt}}_{\text{Effective hours per worker}}.$$

Effective hours per worker equal the productivity-weighted power mean of hours worked:

$$H_{jt} = \left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} \frac{x h^{\frac{\sigma_h-1}{\sigma_h}}\, d\mu^j(h,x)}{\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\,d\mu^j(h,x)}\right)^{\frac{\sigma_h}{\sigma_h-1}}.$$

Intermediate-goods producers choose output y_{jt}, labor input n_{jt}, workforce composition $\{\mu^j(h,x)\}_{h\in\mathcal{H},x\in\mathcal{X}}$, and price p_{jt} to maximize the present discounted value of dividends d_{jt}. Dividends are discounted using the ex-ante real rate of return on equity r_t^e.[22] Price setting is subject to a quadratic price adjustment cost,

$$\frac{\psi}{2}\left[\log\left(\frac{p_{jt}}{p_{j,t-1}}\right)\right]^2 Y_t,$$

as in Rotemberg (1982), where ψ regulates the size of the adjustment cost. The recursive firm problem is:

$$\begin{aligned}
J_t(p_{j,t-1}) &= \max_{y_{jt},n_{jt},\{\mu^j(h,x)\},p_{jt}} d_{jt} + \frac{1}{1+r_t^e} J_{t+1}(p_{jt}) \\
\text{s.t. } d_{jt} &= \frac{p_{jt}}{P_t} y_{jt} - \int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} w(h,x) h \mu^j(h,x)\,dh\,dx - \frac{\psi}{2}\left[\log\left(\frac{p_{jt}}{p_{j,t-1}}\right)\right]^2 Y_t, \\
y_{jt} &= \left(\frac{p_{jt}}{P_t}\right)^{-\varepsilon_p} Y_t, \\
y_{jt} &= z n_{jt} - f, \\
n_{jt} &= \left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\mu^j(h,x) h^{\frac{\sigma_h-1}{\sigma_h}}\,dh\,dx\right)^{\frac{\sigma_h}{\sigma_h-1}} \left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\mu^j(h,x)\,dh\,dx\right)^{-\frac{1}{\sigma_h-1}},
\end{aligned}$$

where $w(h,x)$ denotes the hourly wage rate offered to a worker with productivity x and hours h.

[21] Appendix C.4 discusses the properties of the production function in greater detail.

[22] Under certainty equivalence, discounting with the ex-ante real rate is equivalent to discounting with the stochastic discount factor (SDF).

Optimality conditions are symmetric across firms. I focus on a symmetric-price equilibrium with $p_{jt} = P_t$ for all firms j and periods t. Combining the optimality conditions with symmetric prices yields the New Keynesian Phillips curve (NKPC):

$$\log(1+\pi_t) = \kappa_p \left(mc_t - \frac{\varepsilon_p - 1}{\varepsilon_p} \right) + \frac{1}{1+r_t^e} \frac{Y_{t+1}}{Y_t} \log\left(1+\pi_{t+1}\right),$$

where π_t denotes the inflation rate between periods $t-1$ and t, $\kappa_p = \varepsilon_p/\psi$ is the slope of the Phillips curve, and mc_t is the real marginal cost, with the aggregate wage index $\tilde{w}_t = mc_t z$, and Y_t denotes aggregate output. The NKPC thus takes the functional form familiar from the sticky-price model of Auclert et al. (2023). Since production of each firm j is determined by the CES demand schedule, symmetric prices imply that $y_{jt} = Y_t$ for all firms.

Combining the firm's optimality conditions for labor input n_{jt} and type-specific employee measures $\{\mu^j(h,x)\}$ yields the wage-hours menu offered to workers:

$$w_t(h,x,H_{jt}) = \tilde{w}_t x H_{jt} \left(\frac{h^{\rho-1}}{\rho H_{jt}^{\rho}} + \frac{1}{h}\left(1 - \frac{1}{\rho}\right) \right), \tag{20}$$

where $\rho \equiv \frac{\sigma_h - 1}{\sigma_h}$ and H_{jt} denotes firm-level effective hours per worker. I refer to H_{jt} as reference hours because the hourly wage rate is the highest at these hours. Under CES technology, reference hours H_{jt} are a sufficient statistic for how individual wages depend on the firm's hours distribution.[23] Equation (20) provides a general-equilibrium microfoundation for the wage-hours relationship $f(h,H)$:

$$f(h,H) = H \left(\frac{h^{\rho-1}}{\rho H^{\rho}} + \frac{1}{h}\left(1 - \frac{1}{\rho}\right) \right).$$

I focus on a symmetric pooling equilibrium in which all firms hire the same workforce composition and reference hours are uniform across firms, $H_{jt} = H_t$. The pooling equilibrium follows on both empirical and theoretical grounds. Existing empirical evidence shows substantial within-firm dispersion in hours alongside within-worker hours variation, supporting pooling over separating equilibria (Labanca and Pozzoli, 2022; Shao et al., 2023). Furthermore, labor market frictions prevent firms from targeting specific worker types. Shao et al. (2026) find that even small hiring frictions lead to symmetric equilibria. Specifically, if firms meet workers randomly, a small fixed cost of discarding a worker can sustain the symmetric equilibrium, and workers are paid their marginal product.

[23] Power means are increasing functions of the parameter ρ. In economic terms, this means that with dispersion in hours, the stronger the degree of complementarity in production, the lower the reference hours. Appendix C.5 derives the firm's optimality conditions.

The assumption of a symmetric equilibrium has three major implications for the household and firm blocks of the model. First, all workers face the same wage-hours schedule, with $H_{jt} = H_t$ for every firm

$$w_t(h, x, H_t) = \tilde{w}_t x H_t \left(\frac{h^{\rho-1}}{\rho H_t^{\rho}} + \frac{1}{h}\left(1 - \frac{1}{\rho}\right)\right). \tag{21}$$

Second, all firms have the same workforce size and composition, and output aggregates to

$$Y_t = zN_t - F. \tag{22}$$

Third, the total wage bill that determines firm dividends is a product of the aggregate wage index $\tilde{w}_t$ and aggregate labor N_t:[24]

$$d_t = Y_t - \tilde{w}_t N_t - \frac{\varepsilon_p}{2\kappa_p} \log(1 + \pi_t)^2 Y_t. \tag{23}$$

3.2. Households

The household problem shares the general structure with the corresponding problem introduced in Section 2. I introduce time indices to accommodate time-varying prices along the transition. The earnings $y_t(h_{it}, x_{it}, H_t)$ and hourly wage $w_t(h_{it}, x_{it}, H_t)$ functions come from the solution to the firm's optimization problem. I consider a unit mass of infinitely lived, single-member households. They choose consumption c_{it}, hours worked h_{it}, and savings $a_{i,t+1}$ to maximize the expected value of per-period utility flows from consumption and hours $U(c_{it}, h_{it})$ subject to budget and borrowing constraints:

$$V_t(a_{it}, x_{it}) = \max_{c_{it}, h_{it}, a_{i,t+1}} U(c_{it}, h_{it}) + \beta \mathbb{E}\left[V_{t+1}(a_{i,t+1}, x_{i,t+1}) \mid x_{it}\right] \tag{24}$$

$$\text{s.t.} \quad c_{it} + a_{i,t+1} = (1 + r_t)a_{it} + \underbrace{(1 - \tau_t^w) w_t(h_{it}, x_{it}, H_t) h_{it}}_{y_t(h_{it}, x_{it}, H_t)} + T,$$

$$a_{i,t+1} \geq \underline{a}.$$

The first-order conditions with respect to consumption, labor supply, and next-period asset holdings are

$$U_c(c_{it}, h_{it}) = \lambda_{it}, \tag{25}$$

$$-U_h(c_{it}, h_{it}) = \lambda_{it} y_{h,t}(h_{it}, x_{it}, H_t), \tag{26}$$

$$\beta \mathbb{E}\left[V_{a,t+1}(a_{i,t+1}, x_{i,t+1}) \mid x_{it}\right] \leq \lambda_{it}, \tag{27}$$

[24] I derive the result formally in Appendix C.8.

with equality in (27) whenever the borrowing constraint is slack.[25] The envelope condition with respect to current assets a_{it} is given by

$$V_{a,t}(a_{it}, x_{it}) = (1 + r_t)\lambda_{it}. \tag{28}$$

Following Auclert et al. (2023), I specify the per-period utility functions with a GHH-Plus functional form

$$U(c, h) = \frac{1}{1-\sigma}\left(c - \varphi\alpha\frac{h^{1+\nu}}{1+\nu}\right)^{1-\sigma} - \varphi(1-\alpha)\frac{h^{1+\nu}}{1+\nu}, \tag{29}$$

where ν and σ respectively regulate the curvature of the utility function with respect to hours and with respect to the consumption-hours composite, and α regulates the degree of complementarity between consumption and hours, as measured by the complementarity index introduced in Section 2:

$$\text{CI} = \frac{U_{ch}U_c}{U_{cc}U_h} = \frac{\alpha U_c}{\alpha U_c + 1 - \alpha}.$$

3.3. Asset Market

There are two assets, bonds and equity, and no arbitrage opportunities. The ex-ante real rate of return on bonds, $1 + r_t^e$, equals the expected real rate of return on equity. The expected real rate of return on equity equals the expected dividend yield d_{t+1}/p_t plus the equity gain p_{t+1}/p_t, where p_t denotes the ex-dividend market price of firm equity.

$$1 + r_t^e = \mathbb{E}_t\left[\frac{d_{t+1} + p_{t+1}}{p_t}\right]. \tag{30}$$

The ex-post rate of return on the portfolio r_t is equal to the weighted average of the realized rates of return to each asset, where B denotes the market value of the bond stock[26]

$$1 + r_t = \underbrace{\frac{p_{t-1}}{p_{t-1} + B_{t-1}}}_{\text{equity share}}\frac{d_t + p_t}{p_{t-1}} + \underbrace{\frac{B_{t-1}}{p_{t-1} + B_{t-1}}}_{\text{bond share}}\left(1 + r_{t-1}^e\right).$$

Bonds are real and pay the ex-ante real rate of return, r_{t-1}^e, expected when they are bought.

[25] Because hours choice is continuous without an extensive margin, households choosing very low hours could theoretically face negative labor earnings. In the calibrated model, the fiscal transfer T is sufficiently large to maintain positive consumption, and in the calibrated model earnings are negative only for a negligible fraction of agents.

[26] Under certainty equivalence, households are indifferent between different portfolio risk profiles. Following Auclert et al. (2023), I assume that all households hold the same portfolio of equity and bonds.

3.4. Government

Government spending G_t is discretionary and wasteful. Linear labor taxation τ_t^w is the only source of government revenue. Debt issuance covers the remaining financing needs. Lump-sum transfers T are constant over time. The government budget constraint holds every period:

$$B_t + \tau_t^w \tilde{w}_t N_t = (1 + r_{t-1}^e) B_{t-1} + G_t + T. \tag{31}$$

The deficit financing rule states that whenever government spending increases above its steady-state value, the fraction ρ_B of the extra spending is initially covered by debt issuance. The parameter ρ_B also regulates how fast debt is repaid

$$B_t - B_{ss} = \rho_B \left(B_{t-1} - B_{ss} + G_t - G_{ss} \right).$$

The tax rate τ_t^w varies over time, ensuring the government budget holds even though T is constant, and the deficit financing rule pins down the debt level in each period.

The central bank follows a constant real interest rate rule, with the ex-ante real rate equal to the steady-state real rate $r_t^e = r_{ss}$. The nominal interest rate on bonds, i_t, set by the central bank,

$$i_t = r_{ss} + (1 + r_{ss}) \pi_{t+1},$$

ensures that the ex-ante real rate of return on bonds r_t^e equals the steady-state real interest rate r_{ss}.

3.5. Equilibrium

An equilibrium in this economy consists of a set of decision paths for households $\left\{ c_{it}, h_{it}, a_{i,t+1} \right\}_{i \in [0,1], t=0}^{\infty}$ and firms $\left\{ \{\mu^j(h,x)\}, p_{jt} \right\}_{j \in [0,1], t \in [0,\infty)}$, prices of the final consumption good $\{P_t\}_{t=0}^{\infty}$, the aggregate wage index $\{\tilde{w}_t\}_{t=0}^{\infty}$, the ex-ante real interest rate $\{r_t^e\}_{t=0}^{\infty}$, the realized real interest rate $\{r_t\}_{t=0}^{\infty}$, the equity price $\{p_t\}_{t=0}^{\infty}$, the fiscal variables $\{\tau_t^w, G_t, B_t\}_{t=0}^{\infty}$, the constant transfer T, and the distribution of households over asset and idiosyncratic productivity states $\{D_t(a,x)\}_{t=0}^{\infty}$ such that for every t:

(i) The decision paths for households and firms maximize their objective functions subject to their respective constraints, with prices, taxes, and transfers taken as given.

(ii) The household distribution is consistent with the individual policy functions.

(iii) The government budget constraint (31) holds in all periods.

(iv) The asset, labor, and goods markets clear for all t. In particular, aggregate labor N_t^d equals the

aggregate labor supply N_t^s, and the reference hours H_t^d households take as given are consistent with reference hours H_t^s implied by their choices.[27]

$$
\begin{aligned}
&A_t^d = \int_{x\in\mathcal{X}} \int_{a\in\mathcal{A}} a_{t+1}(a,x) D_t(a,x)\, da\, dx = A_t^s = B_t + p_t \\
&\underset{h,x}{\forall} \quad \mu_t(h,x) = \int_{a\in\mathcal{A}} \mathbb{1}\left\{h(a,x) = h\right\} D_t(a,x)\, da \\
&H_t^d = H_t^s = \left(\int_{x\in\mathcal{X}} \int_{a\in\mathcal{A}} x h_t(a,x)^{\rho} D_t(a,x)\, da\, dx \right)^{\frac{1}{\rho}} \\
&N_t^d = \int n_{jt} dj = N_t^s = H_t^s \int_{x\in\mathcal{X}} \int_{a\in\mathcal{A}} x D_t(a,x)\, da\, dx \\
&Y_t^d = C_t + G_t + \frac{\varepsilon_p}{2\kappa_p} \log\left(1+\pi_t\right)^2 Y_t = Y_t^s = zN_t - F
\end{aligned}
$$

4. Calibration

To discipline the fiscal policy analysis, I calibrate the model to match a quarterly MPC of 0.25 and a set of moments characterizing asset supply, taxes, and transfers in the United States. On the household side, the objective is to generate realistic consumption responses to unanticipated transfers, liquid asset demand, and dispersion in labor income. To maximize comparability with the baseline sticky-price model subject to the HANK trilemma, I calibrate the model to the same targets as in Auclert et al. (2023). I cast the model in quarterly frequency.

I calibrate the model in two steps. First, I externally calibrate the parameters characterizing fiscal policy, the New Keynesian block, and the Frisch elasticities of labor supply, and summarize them in Table 1. Second, I calibrate the remaining parameters characterizing the household block, aggregate hours, and output internally, which I display in Table 2. Imperfect substitutability between hours worked in production is the central theoretical innovation compared to the standard sticky-price HANK model. I review the empirical evidence about hours complementarities and detail the exact mapping to the Frisch elasticities in the dedicated subsection below.

4.1. Externally Calibrated Parameters

Government

I set the labor income tax rate to $\tau^w = 0.334$, which captures income taxes and social security contributions in the US. Lump-sum transfers T equal 14.3% of aggregate pre-tax labor

[27] Since everybody works and average idiosyncratic productivity is one in the calibrated model, reference hours equal aggregate hours and the consistency conditions are jointly satisfied.

earnings $\tilde{w}N$.[28] In the steady state, the government supplies bonds worth 55% of annual GDP, and the total firm equity value equals 85% of annual GDP. The quarterly real interest rate equals 0.5%. Government debt is persistent, with an AR(1) parameter of $\rho_B = 0.9$. The government calibration implies a steady-state transfers-to-GDP ratio of $\frac{T}{Y} = 0.1406$ and a government-spending-to-GDP ratio of $\frac{G}{Y} = 0.1768$.

The New Keynesian Phillips Curve

I set the elasticity of substitution between intermediate varieties to $\varepsilon_p = 7$, implying a steady-state markup of $\mu = 1.1667$. I set the slope of the New Keynesian Phillips curve for prices to $\kappa_p = 0.01$.

Table 1: Externally calibrated parameters

Parameter	Description	Value	Source
ν^{-1}	aggregate Frisch elasticity	0.5	Battisti et al. (2024)
τ^w	wage tax rate	0.334	Auclert and Rognlie (2020)
T	transfer	$0.143 \cdot \tilde{w}N$	Auclert and Rognlie (2020)
ε_p	elasticity of substitution	7	Auclert et al. (2023)
κ_p	NKPC slope	0.01	Auclert et al. (2023)
B	government bonds	$0.55 \cdot 4Y$	Auclert et al. (2023)
r	quarterly real rate	0.5	Auclert et al. (2023)
ρ_B	persistence of public debt	0.9	Auclert et al. (2024)
$\rho_{\log x}$	income shock persistence	0.966	McKay et al. (2016)

Income Process

I assume that idiosyncratic productivity x follows an AR(1) process in logs with persistence $\rho_{\log x}$

$$\log x' = (1 - \rho_{\log x})\mu_{\log x} + \rho_{\log x} \log x + \varepsilon_{\log x}.$$

[28] Auclert and Rognlie (2020) show that the relationship between average income plus transfers net of taxes and average pre-tax income is approximately linear. Their estimates imply $\tau^w = 0.334$ and $T = 0.143 \cdot \tilde{w}N$.

I set the mean of the income process $\mu_{\log x}$ to normalize the average idiosyncratic productivity

$$\mathbb{E}[x] = \exp\left(\mu_{\log x} + \frac{\sigma^2_{\log x}}{2}\right) = 1.$$

I set the AR(1) persistence parameter to $\rho_{\log x} = 0.966$, and discretize the income process into a Markov chain using the Rouwenhorst method. I calibrate the variance of log idiosyncratic productivity internally, which I discuss in the dedicated subsection.

The Idiosyncratic and Aggregate Frisch Elasticities of Labor Supply

The distinction between the idiosyncratic and aggregate Frisch elasticities is a key property of hours complementarities, allowing the model to generate substantially lower MPEs without contradicting the micro evidence on Frisch elasticities. Two parameters govern these elasticities in my model: the hours complementarity parameter ρ and the utility curvature parameter ν. I consider a range of values for ρ, including estimates from the literature. Shao et al. (2023) find an elasticity of substitution between hours of $\sigma_h = 0.685$ ($\rho = -0.46$) using Canadian matched employer-employee data. Identification comes from within-firm variation in hours and wages. Battisti et al. (2024) estimate a structural labor supply model with potential complementarities in production using matched employer-employee data from the Italian region of Veneto, and find that hours are gross complements in production, with an estimated elasticity of substitution between hours of $\sigma_h = 0.338$ ($\rho = -1.962$).

The distinction between the two elasticities is transparent from inspecting the optimal consumption-hours choice under hours complementarities. Substituting the wage-hours profile offered by firms into the household's optimal labor supply condition (26) yields:

$$-U_h(c_{it}, h_{it}) = \lambda_{it}\tilde{w}^{\tau,x}_{it}\left(\frac{h_{it}}{H_t}\right)^{\rho-1}, \tag{32}$$

where the after-tax wage factor $\tilde{w}^{\tau,x}_{it} \equiv \tilde{w}^{\tau}_{t}x_{it}$ is defined as in Section 2.

Taking logs and differentiating with respect to $\tilde{w}^{\tau,x}_{it}$ yields the following expression for the idiosyncratic Frisch elasticity:

$$\text{Frisch}_i \equiv \frac{\partial \log h(\lambda, \tilde{w}^{\tau,x}, H)}{\partial \log \tilde{w}^{\tau,x}} = \frac{1}{1-\rho+\nu}, \tag{33}$$

where $\nu \equiv U_{hh}h/U_h - U^2_{ch}h/U_{cc}/U_h$.[29] The idiosyncratic Frisch elasticity measures the responsiveness of hours worked to temporary wage changes holding the marginal utility of consumption

[29] Under GHH-Plus preferences this coincides with the utility exponent ν, since $U_{hh} - U^2_{ch}/U_{cc} = -\varphi\nu h^{\nu-1}(\alpha z^{-\sigma} + 1 - \alpha)$ with $z = c - \varphi\alpha h^{1+\nu}/(1+\nu)$.

and reference hours H_t fixed. In contrast, the aggregate Frisch elasticity, as defined in Section 2, measures how aggregate hours H respond to changes in the after-tax wage index[30]

$$\text{Frisch}_a \equiv \left.\frac{\partial \log H_t}{\partial \log \tilde{w}_t^{\tau}}\right|_{\{\lambda_{it}\}=\bar{\lambda}_i} = \frac{1}{\nu}. \tag{34}$$

Battisti et al. (2024) also provide estimates that distinguish between the labor supply responsiveness to idiosyncratic and firm-level shocks, which inform the idiosyncratic and aggregate Frisch elasticities in my model.[31] They estimate the curvature parameter of the disutility function ν, and recover $\nu^{-1} = 0.483$. I set ν to 2 as in Auclert et al. (2023), which implies an aggregate Frisch elasticity of 0.5.[32] Given ν, the idiosyncratic Frisch elasticity depends on the degree of complementarity between hours as regulated by the complementarity parameter ρ. The idiosyncratic Frisch elasticities implied by the estimates of Shao et al. (2023) and Battisti et al. (2024) are 0.29 and 0.2, respectively.

4.2. Internally Calibrated Parameters

Production and Dividends

Adjusting the fixed cost in production is a convenient tool to target equity value without compromising the markup calibration. In steady state, firm dividends satisfy

$$d = zN - \underbrace{\frac{z}{\mu}}_{\tilde{w}} N - F = \left(1 - \frac{1}{\mu}\right) zN - F.$$

The per-period return to all equity holdings equals total dividends,

$$pr = d = \left(1 - \frac{1}{\mu}\right) zN - F. \tag{35}$$

In steady state, no arbitrage dictates that the rate of return on equity equals the real rate of return on bonds ($r = 0.005$). I normalize aggregate labor to unity, $N = 1$. The remaining two parameters, F and z, jointly determine total output and total equity value. Normalizing output to unity and

[30] Appendix B.1 derives the formula for the aggregate Frisch elasticity by leveraging that optimal hours are closed-form under GHH-Plus preferences.

[31] Aggregate wage shocks affect all workers in the firm, so aggregate hours respond in the same way as reference hours in each firm.

[32] The intensive-margin aggregate Frisch elasticity of around 0.5 is slightly below the labor supply elasticity estimates based on tax and retirement system changes reported by Chetty et al. (2013).

substituting the production function relation $F = zN - 1$ into equation (35) yields a closed-form expression for z,

$$z = \frac{\mu\,(1 - pr)}{N}.$$

The fixed production cost F is determined residually,

$$F = zN - 1,$$

ensuring that firm dividends yield an equity value p equal to 0.85 times annual GDP (Auclert et al., 2023).

Income Process

I calibrate the variance of log idiosyncratic productivity, $\sigma^2_{\log x} = \frac{\sigma^2_{\varepsilon_{\log x}}}{1-\rho^2_{\log x}}$, to match a cross-sectional standard deviation of log labor earnings of 0.92 (Song et al., 2018).[33] With hours complementarities, labor earnings are negative at low hours, which affects the calibration of the income process. In the standard model, labor earnings are linear in hours: $y(h, \tilde{w}^{\tau,x}) = \tilde{w}^{\tau,x} h$. However, under imperfect substitutability, the limit of labor earnings at zero hours is negative:

$$\lim_{h\to 0^+} y(h, \tilde{w}^{\tau,x}, H) = \lim_{h\to 0^+} \tilde{w}^{\tau,x} H \left(\frac{1}{\rho}\left(\frac{h}{H}\right)^{\rho} + 1 - \frac{1}{\rho}\right) < 0.$$

Because negative values are undefined in logs, I calibrate the variance of log labor earnings using only households with positive earnings:

$$\text{Var}\left[\log y(h, \tilde{w}^{\tau,x}, H) \mid y(h, \tilde{w}^{\tau,x}, H) > 0\right].$$

As I show in Appendix C.11, less than 0.04% of simulated households exhibit negative labor earnings across all parameterizations.

Preferences

To match a quarterly MPC of 0.25 and liquid asset demand in the data, I follow Auclert et al. (2023) and introduce heterogeneous discount factors.[34] Half of the household population

[33] I report the calibrated values of $\sigma_{\log x}$ in Appendix C.10.

[34] Kaplan and Violante (2022) group the evidence on the MPCs into quasi-experimental studies (Johnson et al., 2006; Parker et al., 2013; Misra and Surico, 2014; Fagereng et al., 2021), studies using survey instruments (Jappelli and Pistaferri, 2014; Fuster et al., 2021), and studies using semi-structural methods (Blundell et al., 2008; Commault, 2022), among many others. They conclude that the MPC on nondurable goods and services is between 15% and 25%. However, the most recent state-of-the-art evidence suggests lower MPCs between 0 and 0.1 (Borusyak et al., 2024; Orchard et al., 2025; Boehm et al., 2025). With these lower values, the HANK trilemma disappears or is easier to solve.

Table 2: Internally-calibrated Parameters

Parameter	Description	Value	Target
β_u	upper discount factor	Table C.2	$A = 1.4 \cdot 4Y$
β_l	lower discount factor	Table C.3	MPC = 0.25
$1/\sigma$	U_c curvature	Table C.4	average EIS = 0.5
φ	disutility of labor	Table C.5	$H = 1$
z	aggregate productivity	1.146833	$Y = 1$
$\sigma_{\log x}$	sd of log idiosyncratic productivity	Table C.6	$\text{Var}[\log y] = 0.92^2$
F	fixed cost	0.146833	$p = 0.85 \cdot 4Y$

Note: The inner loop solves for all parameters except β_l to 9-digit precision. The outer loop performs bisection to find β_l that matches the MPC. Since MPC is not a continuous function of this parameter, the fit is not exact. Tables C.2 to C.6 present the calibrated parameters.

is assigned a high discount factor, and the other half a low discount factor. I calibrate the high discount factor to match aggregate liquid asset demand of 1.4 times annual GDP (McKay et al., 2016), and the low discount factor to match the aggregate quarterly MPC of 0.25.

Table 3: MPCs across preferences and hours complementarities in production

	α										
ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.251	0.250	0.251	0.258	0.250	0.249	0.249	0.250	0.251	0.250	0.249
-0.46	0.251	0.244	0.249	0.250	0.251	0.249	0.250	0.251	0.250	0.250	0.250
-1.962	0.251	0.250	0.251	0.251	0.250	0.250	0.249	0.249	0.250	0.250	0.250
-7	0.250	0.250	0.251	0.249	0.250	0.251	0.250	0.250	0.251	0.250	0.250

Note: The parameter α controls the degree of complementarity between consumption and hours, with $\alpha = 0$ corresponding to separable preferences and $\alpha = 1.0$ corresponding to GHH preferences. The parameter ρ controls complementarities between hours worked, with $\rho = 1.0$ corresponding to a standard linear aggregation, where hours are perfect substitutes, and values below zero indicating that hours are gross complements.

I calibrate the utility curvature parameter σ to match the average elasticity of intertemporal substitution (EIS) of 0.5.[35] I follow Auclert et al. (2023) and consider a range of values for the

[35]Appendix C.3 details how consumption-hours complementarities in preferences affect the EIS calculation.

parameter α that regulates complementarity between consumption and hours in preferences.

Additionally, I calibrate the labor disutility parameter φ such that aggregate labor supply N^s equals labor demand $N^d = 1$, ensuring labor market clearing. Since the calibration of the income process ensures that the average idiosyncratic productivity equals one, aggregate labor equals reference hours

$$N_t = H_t.$$

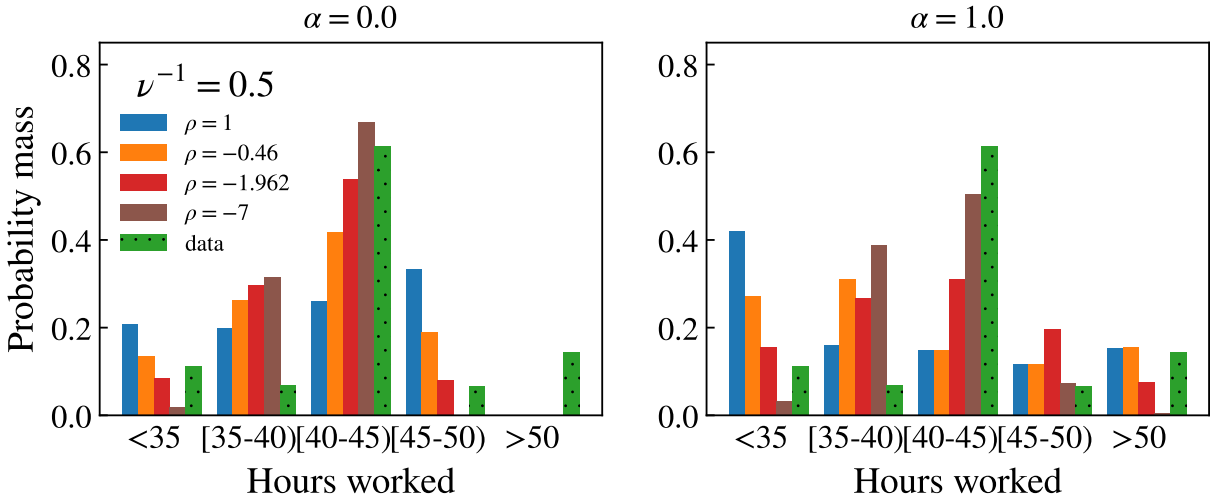
4.3. The Model's Fit to the Data

Targeted Moments

All model calibrations match all targeted moments exactly except the MPC, which is matched approximately because the MPC is discontinuous in the discount factor. I report the calibrated MPCs in Table 3. For most model calibrations, the fit to a quarterly MPC of 0.25 is excellent.

Untargeted Moments

Figure 1: Hours distribution in the models and data



Note: The left-hand and right-hand panels present the steady-state hours distributions in the models with separable ($\alpha = 0.0$) and GHH ($\alpha = 1.0$) preferences and in the data. Different values of parameter ρ correspond to different degrees of hours complementarities in production. The lower the ρ, the more complementary hours worked are in production. Data source: CPS ORG, September 1995-December 2019 for women and men.

As Figure 1 shows, hours complementarities improve the model's fit to the fraction of full-

Table 4: Standard deviation of log hours across preferences and hours complementarities

	α										
ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	0.22	0.17	0.18	0.19	0.21	0.23	0.25	0.26	0.28	0.29	0.31
-0.46	0.14	0.11	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19
-1.962	0.10	0.08	0.08	0.09	0.10	0.10	0.11	0.12	0.13	0.13	0.14
-7	0.05	0.04	0.04	0.05	0.05	0.06	0.06	0.07	0.07	0.07	0.08
Data						0.25					

Note: The parameter α controls the degree of complementarity between consumption and hours, with $\alpha = 0$ corresponding to separable preferences and $\alpha = 1.0$ corresponding to GHH preferences. The parameter ρ controls complementarities between hours worked, with $\rho = 1.0$ corresponding to a standard linear aggregation, where hours are perfect substitutes, and values below zero indicating that hours are gross complements. I calculate the standard deviation of log hours in the data using a sample of working-age women and men who are not self-employed in CPS ORG 09.1995-12.2019.

time workers, but worsen the fit to the tails of the distribution.[36] Hours complementarities lower the idiosyncratic Frisch elasticity and yield a more concentrated distribution of hours worked, with the fraction of workers at modal hours closer to its empirical counterpart.[37] However, the fraction of households working fewer than 35 or more than 50 hours declines dramatically, dropping well below the empirical levels. Relatedly, as I report in Table 4, the standard deviation of log hours drops dramatically, falling well below the levels observed in the data. It is a well-established fact in the literature that matching the distribution of hours worked requires a combination of wage-hours dependence and preference heterogeneity (Bick et al., 2022).[38]

5. The HANK Trilemma with Hours Complementarities

I use the calibrated model to answer the sticky-price leg of my research question: Can complementarities between workers' hours in production solve the HANK trilemma? I compute the MPE and present-value fiscal multiplier in models with varying degrees of nonseparability in

[36]To compare model and empirical hours, I assume the model average hours correspond to the average hours in the data. See Appendix C.12 for further details.

[37]In Appendix C.13, I show that this effect is more pronounced at lower values of the complementarity index, for example, for separable preferences.

[38]Moreover, as reported in Appendix C.14, hours complementarities lead to a more equal distribution of wealth.

preferences and with varying degrees of hours complementarity in production. All models are calibrated to match the quarterly MPC of 0.25.

For comparability with Auclert et al. (2023), I set the target range for the MPE between 0 and 0.04 and for the fiscal multiplier between 0.6 and 2.0. The MPE range covers the point estimate of 0.023 from Golosov et al. (2024). The empirical range for the fiscal multiplier comes from the review article by Ramey (2011). However, more recent evidence points towards fiscal multipliers no higher than 1.5 under accommodative monetary policy (Ramey and Zubairy, 2018; Ramey, 2019).

5.1. The Computational Approach

To compute the MPE, I study partial-equilibrium transitional dynamics of the household block in response to a small unanticipated transfer shock that leaves reference hours H_t unchanged. Since the model is quarterly, I compute the annual MPE as a ratio of the sum of the cross-sectional average labor earnings responses over the first four quarters to the one-time transfer shock.[39]

To calculate the present-value fiscal multiplier, I compute the general-equilibrium first-order responses to an MIT government spending shock of persistence $\rho_G = 0.9$ using the sequence-space Jacobian approach. Specifically, I compute the Jacobian of the household block using the fake-news algorithm, and the Jacobians of the remaining blocks using first differences. I apply the chain rule along the model's directed acyclic graph to obtain the derivatives of model outcomes with respect to the shock and the endogenous variables. I then use the implicit function theorem to recover the first-order output responses to the government spending shock.

5.2. HANK Trilemma under Hours Complementarities in Production

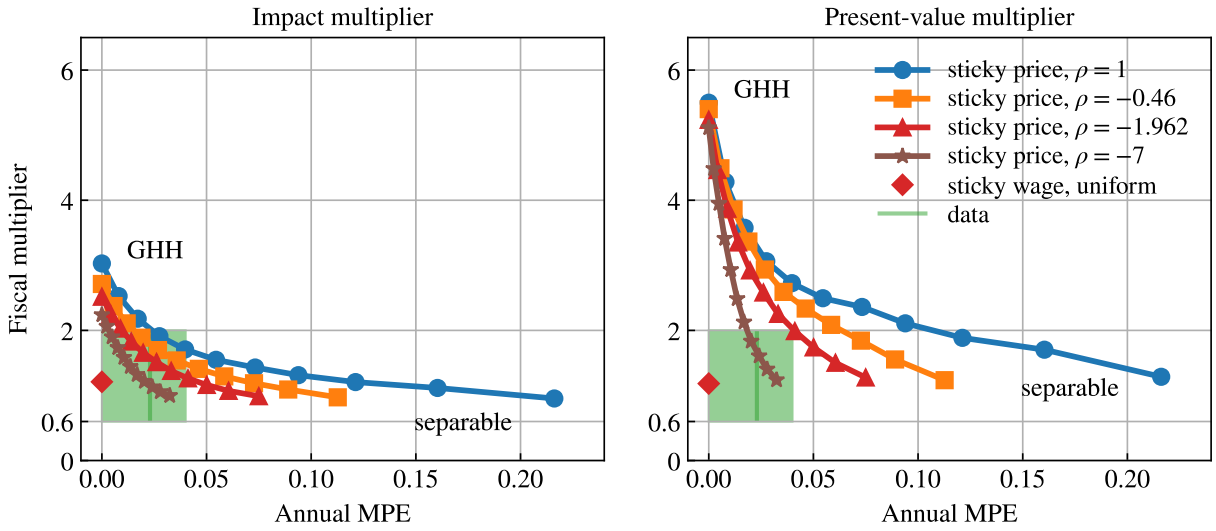
I investigate how varying the strength of hours complementarities in production affects the MPE–fiscal multiplier trade-off. To do so, I compute the MPE and fiscal multipliers for models with different degrees of complementarity between consumption and hours in preferences, as regulated by α, and varying degrees of hours complementarities in production, as regulated by ρ.

Figure 2 shows that hours complementarities in production substantially reduce MPEs across all specifications while having a negligible effect on the fiscal multiplier. The blue dots

[39] Appendix C.6 outlines the procedure to calculate the model MPE and how it corresponds to empirical evidence. In particular, I discuss the tension between matching low annual MPEs and high lifetime MPEs out of unearned income reported in Golosov et al. (2024).

correspond to the baseline sticky-price model of Auclert et al. (2023). With separable preferences, the baseline model yields an MPE of 0.216, well above the target range. With GHH preferences, fiscal multipliers in the baseline model explode. However, as I decrease ρ and thus decrease the elasticity of substitution between individual hours worked in production, σ_h, the MPEs decline substantially across all models. Graphically, the improvement corresponds to a leftward shift of the orange, red, and brown dotted lines relative to the blue line.

Figure 2: MPEs and fiscal multipliers in the sticky-price models with hours complementarities



Note: The left-hand panel presents the impact multiplier, the ratio of the instantaneous output response to the change in government spending, and the right-hand panel presents the present-value fiscal multiplier, the ratio of the present values of the output and government spending changes. Different colors correspond to different degrees of hours complementarities in production, which the parameter ρ regulates. The values $\rho = -0.46$ and $\rho = -1.962$ correspond to the estimates of Shao et al. (2023) and Battisti et al. (2024). The baseline model with linear aggregation corresponds to $\rho = 1$, and values below zero indicate that hours are gross complements in production. Different dots of the same color correspond to different degrees of complementarity between consumption and hours in preferences, from separable preferences on the right to GHH preferences on the left. The green rectangle is the target range of Auclert et al. (2023) for the MPE and the fiscal multiplier; the vertical green line inside the box is the point MPE estimate of 0.023 from Golosov et al. (2024).

Complementarities between hours worked in the range of micro estimates close half to three-quarters of the gap between the baseline MPE and the data. Existing empirical evidence shows that hours are gross but imperfect complements in production, which disciplines the range of MPEs that models with hours complementarities can support. The lowest available estimate of the elasticity of substitution between hours worked in production is $\sigma_h = 0.338$ (Battisti et al., 2024), which implies an MPE of 0.075 with separable preferences. Shao et al. (2023) estimate an elasticity of substitution between hours of $\sigma_h = 0.685$ in Canada, which implies an MPE of 0.113. As I report in Table 5, a higher degree of complementarity with an elasticity of substitution

Table 5: MPEs across preferences and hours complementarities

	α					
ρ	0	0.2	0.4	0.6	0.8	1
1	0.216	0.121	0.073	0.040	0.017	0.000
-0.46	0.113	0.073	0.046	0.027	0.012	0.000
-1.962	0.075	0.050	0.033	0.020	0.009	0.000
-7	0.032	0.024	0.017	0.011	0.005	0.000

Note: For each model, I compute annual MPEs from partial-equilibrium transitional dynamics of after-tax labor earnings to an unanticipated transfer shock. The parameter α controls the degree of complementarity between consumption and hours, with $\alpha = 0$ corresponding to separable preferences and $\alpha = 1.0$ corresponding to GHH preferences. The parameter ρ controls complementarities between hours worked, with $\rho = 1.0$ corresponding to a standard linear aggregation, where hours are perfect substitutes, and values below zero indicating that hours are gross complements.

Table 6: Present-value fiscal multipliers in models with different α and ρ parameters

	α					
ρ	0	0.2	0.4	0.6	0.8	1
1	1.287	1.886	2.360	2.726	3.577	5.499
-0.46	1.235	1.843	2.336	2.936	3.858	5.400
-1.962	1.276	1.739	2.253	2.922	3.870	5.236
-7	1.239	1.606	2.127	2.932	3.950	5.113

Note: Monetary policy follows a constant real rate rule. For each model, I compute the present-value fiscal multiplier from first-order general-equilibrium responses of output to a persistent government spending shock. The present-value fiscal multiplier equals the ratio of the present value of changes in output to the present value of changes in government spending. The parameter α controls the degree of complementarity between consumption and hours, with $\alpha = 0$ corresponding to separable preferences and $\alpha = 1.0$ corresponding to GHH preferences. The parameter ρ controls complementarities between hours worked, with $\rho = 1.0$ corresponding to a standard linear aggregation, where hours are perfect substitutes, and values below zero indicating that hours are gross complements.

between hours below $\sigma_h = 0.125$ ($\rho = -7$) is required to match the MPE. The elasticities of substitution of 0.685, 0.338, and 0.125 correspond to idiosyncratic Frisch elasticities of around 0.29, 0.2, and 0.1.

The canonical sticky-wage model relies on the uniform hours assumption to solve the HANK trilemma. Workers are off their labor supply curves, and the MPE is zero by construction. The fiscal multiplier is moderately sized because preferences are separable, and the complementarity index between consumption and hours is zero.

5.3. Why Do Hours Complementarities Help Solve the HANK Trilemma?

This section builds on the analytical arguments developed in Section 2 to argue that a wedge between the idiosyncratic and aggregate Frisch elasticities allows the model to deliver lower MPEs without compromising the responsiveness of aggregate hours to transitory wage shocks.

Why does the MPE decline? As shown in Section 2, the idiosyncratic Frisch elasticity increases with the elasticity of marginal earnings with respect to hours worked. In the model with imperfect substitutability between hours in production, this elasticity is constant,

$$\frac{y_{hh}h}{y_h} = \rho - 1, \tag{36}$$

and is governed directly by the complementarity parameter ρ. The stronger the complementarities in hours, the quicker the earnings gains from working longer hours dissipate as hours increase.

The improvement in the MPE is thus consistent with Proposition 2, which relates the MPE-MPC ratio to the idiosyncratic Frisch elasticity. MPEs are calculated based on idiosyncratic variation in wealth, which does not move reference hours. As equation (33) shows, hours complementarities reduce the idiosyncratic Frisch elasticity to $1/(1 - \rho + \nu)$ by introducing decreasing returns to hours. A lower idiosyncratic Frisch elasticity translates into a lower MPE. At the same time, the aggregate Frisch elasticity remains $1/\nu$ because the feedback effect from aggregate hours to individual hours offsets the lower idiosyncratic Frisch elasticity, consistent with Proposition 1.

Are the model's Frisch elasticities consistent with the micro evidence? The concepts of idiosyncratic and aggregate Frisch elasticities are new in the context of aggregate labor supply and distinct from the micro and macro Frisch elasticities widely debated in macroeconomics.[40] Micro

[40]The distinction between idiosyncratic and firm-level variation in hours is originally due to Battisti et al. (2024).

elasticities come from quasi-experimental evidence, and macro elasticities come from estimated business-cycle models.[41] Micro estimates of the Frisch elasticity along the intensive margin vary greatly across different estimates in the literature, ranging from 0.025 in a study that uses Swiss tax holidays by Martínez et al. (2021) up to 0.87 for women in Attanasio et al. (2018). The meta-analyses performed by Chetty et al. (2011) and Elminejad et al. (2023) find intensive-margin Frisch elasticities of 0.54 and 0.2, respectively. Typical macro Frisch elasticity estimates range between 2.6 and 4 (Chetty et al., 2013). Micro and macro estimates of the Frisch elasticity differ substantially, and most of the volatility in aggregate hours comes from the extensive margin (Ashenfelter, 1984; Heckman, 1984), which my model does not include.[42] Nonetheless, the aggregate Frisch elasticity is the same as in the baseline model of Auclert et al. (2023).

The model counterpart of the micro Frisch elasticity lies between the idiosyncratic and aggregate Frisch elasticities. The MPE is identified from lottery winnings, an idiosyncratic wealth shock that leaves reference hours in the firm unchanged, so the idiosyncratic Frisch elasticity governs it. Existing Frisch elasticity estimates instead rely on potentially broader wage variation that affects multiple workers in the firm. For my baseline calibration with $\rho = -0.46$, the idiosyncratic Frisch elasticity is 0.29 and the aggregate Frisch elasticity is 0.5, both of which lie within the range of the empirical estimates. The model-implied micro Frisch elasticity therefore does not contradict the data.

Why do the fiscal multipliers not increase? The intratemporal condition (32) provides clear intuition for why hours complementarities do not affect the aggregate Frisch elasticity. Marginal earnings depend on the ratio of individual hours h_{it} to reference hours H_t. Because aggregate shocks move both hours variables in equal proportion, this ratio remains constant, rendering the complementarity parameter ρ irrelevant to aggregate wage variation.

Through the lens of Proposition 1, hours complementarities do not affect the aggregate Frisch elasticity because the feedback effect from aggregate hours offsets the reduction in the

[41] Historically, the macro literature used the term macro elasticity to refer to the Frisch elasticity of aggregate hours and micro elasticity to refer to the intensive-margin elasticity of hours conditional on employment. However, Chetty et al. (2013) show that the traditional split between the micro and macro elasticities conflates two distinct dimensions: the margin of adjustment (intensive versus extensive) and the source of wage variation (temporary versus permanent, and quasi-experimental versus calibrated to aggregate moments). The identification strategy is therefore a more useful criterion for distinguishing between the micro and macro elasticities.

[42] As Chetty et al. (2013) argue, labor supply choices along the extensive margin cannot explain the micro-macro Frisch gap. Theories combining the extensive margin with labor market frictions are good candidates to explain the micro-macro Frisch gap (Hall, 2009).

idiosyncratic Frisch elasticity. The elasticity of marginal earnings with respect to aggregate hours is constant and equal to

$$\epsilon_{y_h,H} = 1 - \rho. \tag{37}$$

Therefore, since both the idiosyncratic Frisch elasticity and the elasticity of marginal earnings to aggregate hours are constant, Proposition 1 implies

$$\text{Frisch}_a = \frac{\text{Frisch}_i}{1 - \text{Frisch}_i \epsilon_{y_h,H}} = \frac{\frac{1}{1-\rho+\nu}}{1 - \frac{1-\rho}{1-\rho+\nu}} = \frac{1}{\nu}.$$

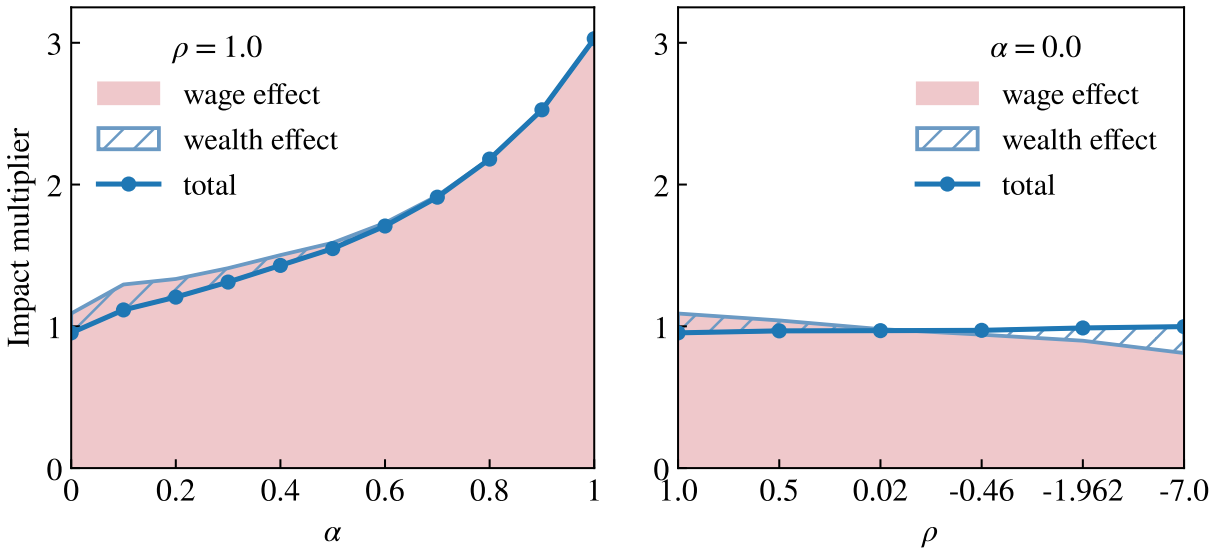
Hours complementarities do not affect the aggregate Frisch elasticity, which I keep constant at $\nu^{-1} = 0.5$ across all models. While the present-value fiscal multiplier is the headline measure, the impact multiplier is easier to study analytically because the distribution is in the steady state at time 0. As I show in Proposition 3, the impact multiplier is proportional to the aggregate Frisch elasticity, which scales both the wage and wealth effects of government spending. I also leverage Proposition 3 to decompose the impact multiplier into the general-equilibrium wage and wealth effects:

$$\frac{dY_0}{dG_0} = \underbrace{\text{Frisch}_a \frac{zH}{G} \Omega_{\tilde{w}^\tau,G}}_{\text{wage effect}} + \underbrace{\text{Frisch}_a \frac{zH}{G} \int \int (1 - \text{CI}(a,x)) \, \Omega_{\lambda,G}(a,x) \, \omega(a,x) \, da \, dx}_{\text{wealth effect}} \tag{38}$$

The complementarity index equals one with GHH preferences, so the wealth effect vanishes and the decomposition attributes the whole impact multiplier to the wage effect at $\alpha = 1$. Figure 3 shows that nonseparable preferences increase the impact multiplier by inflating the general-equilibrium wage effect and reducing the general-equilibrium wealth effect on labor supply.

Hours complementarities reduce the general-equilibrium wage effect and increase the general-equilibrium wealth effect by a similar amount, leaving the multiplier largely unchanged. At the same time, they reduce MPEs, as Table 5 shows. Nonseparable preferences reduce the hours response to wealth changes through the multiplier λ without affecting how hours respond to changes in wages. For high-MPC households, most resources come from labor income. Therefore, shutting down the wealth effect increases hours, disposable income, and thereby spending of these households, driving the multiplier. Hours complementarities, however, affect both margins symmetrically through the idiosyncratic Frisch elasticity, as shown in equation (8). Crucially, the responsiveness of hours to changes in wages and wealth remains balanced, so unlike under nonseparable preferences, fiscal multipliers do not explode.

Figure 3: Impact multiplier decomposition between wage and wealth effects



Note: The left-hand panel presents the impact multiplier, the ratio of the instantaneous output response to the change in government spending, across different degrees of nonseparability between consumption and hours in preferences α, and the right-hand panel does so across different degrees of complementarity between hours worked in production, as regulated by the parameter ρ. The values $\rho = -0.46$ and $\rho = -1.962$ correspond to the estimates of Shao et al. (2023) and Battisti et al. (2024). The baseline model with linear aggregation corresponds to $\rho = 1$, and values below zero indicate that hours are gross complements in production.

5.4. Hours Complementarities and Fiscal Multipliers under Alternative Monetary Policy Regimes

In this section, I compute the general-equilibrium output responses to government spending shocks while constraining the nominal interest rate to its steady-state level for 12 quarters. Specifically, I let monetary policy follow the Taylor rule,

$$i_t = r_{ss} + \phi_t \pi_t,$$

where i_t is the nominal interest rate on government bonds, r_{ss} is the steady-state real interest rate, and ϕ_t is the Taylor coefficient on inflation π_t. Under the three-year interest rate peg, I set ϕ_t to zero in the first 12 quarters of the transition. The goal of the exercise is to compute present-value fiscal multipliers under a strong monetary policy accommodation, mimicking the effects of the zero lower bound. The standard sticky-price HANK model generates fiscal multipliers under a three-year interest rate peg as high as 73.92, almost two orders of magnitude larger than in the data.[43]

[43] Ramey and Zubairy (2018) estimate fiscal multipliers when monetary policy is at the zero lower bound. The results are mixed. Depending on the specification, the difference between fiscal multipliers during the ZLB and non-ZLB

A combination of equity revaluation under sticky prices and labor earnings linear in hours explains why the present-value fiscal multiplier explodes under the three-year nominal interest rate peg, and complementarities in hours curb the latter effect by introducing decreasing returns to hours. First, in sticky-price models, markups and asset income are countercyclical. Second, in the standard sticky-price model, earnings are linear in hours, and households react to the negative asset income effect by working very long hours.[44] Complementarities in hours reduce the opportunities to use labor supply to offset the asset devaluation. Because the wealth effects of a monetary-fiscal expansion are heterogeneous across wealth, reference hours move less than one-for-one with hours of the wealthiest workers. As a result, decreasing returns to hours kick in, and wealthy households adjust their hours worked less than in the baseline model.

Figure 4 illustrates how complementarities curb the expansion of hours by high-wealth households, and Table 7 confirms that fiscal multipliers under the three-year nominal interest rate peg decline as a result. Fiscal multipliers under hours complementarities are more than an order of magnitude lower than in the baseline model, standing at 2.40–2.53 vs. 73.92 and substantially closer to the empirical upper bound of 1.5 (Ramey and Zubairy, 2018). As the second panel of Figure 4 shows, even moderate complementarities between hours worked make the problem far less severe.

The canonical sticky-wage solution generates lower fiscal multipliers under the three-year nominal interest rate peg by assuming uniform hours and making profits procyclical. When wages are sticky and prices are flexible, wages become less procyclical, and profits become procyclical. As a result, the income expansion of hand-to-mouth households and the labor supply expansion of wealthy households are smaller, which dampens fiscal multipliers.

Furthermore, the sticky-wage model implies that fiscal multipliers are higher when debt is more persistent. The sticky-price model, by contrast, inherits from the model of Auclert et al. (2023) the interaction between tradable equity and linear labor taxation, which results in first-order Ricardian equivalence. Appendix D proves formally that hours complementarities do not overturn this result, and the fiscal columns of Table 7 confirm it numerically at every degree of complementarity.

periods is not statistically significant, or fiscal multipliers at the ZLB are higher (1.4 vs. 0.6).

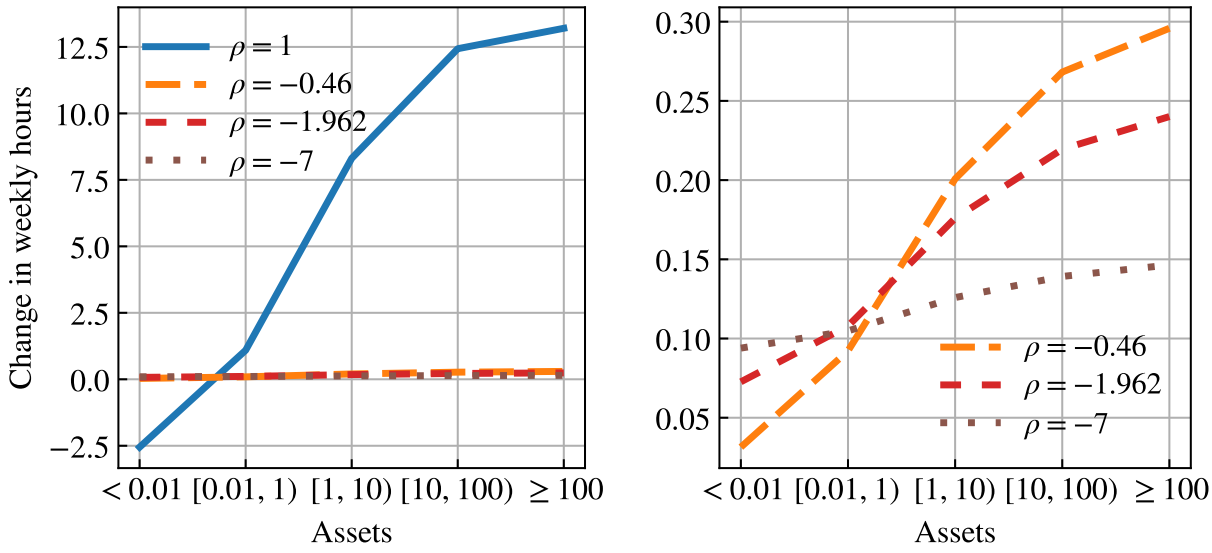
[44]The observation that sticky-price New Keynesian models are sensitive to the interaction between the distribution of asset income and endogenous labor supply goes back to Broer et al. (2020).

Table 7: Present-value multiplier under different monetary and fiscal policy regimes

	Monetary regime			Fiscal regime		
	Taylor rule	Constant-r	3-year peg	$\rho_B = 0$	$\rho_B = 0.9$	$\rho_B = 0.95$
	$\rho = 1$, $\sigma_h \to \infty$, hours are perfect substitutes					
Separable	0.77	1.29	73.92	1.29	1.29	1.29
$\alpha = 0.5$	1.14	2.49	3.49	2.49	2.49	2.49
GHH	2.39	5.50	6.00	5.50	5.50	5.50
	$\rho = -0.46$, $\sigma_h = 0.685$, Shao et al. (2023)					
Separable	0.77	1.24	2.53	1.24	1.24	1.24
$\alpha = 0.5$	1.20	2.59	3.68	2.59	2.59	2.59
GHH	2.27	5.40	6.38	5.40	5.40	5.40
	$\rho = -1.962$, $\sigma_h = 0.338$, Battisti et al. (2024)					
Separable	0.80	1.28	2.40	1.28	1.28	1.28
$\alpha = 0.5$	1.22	2.58	3.69	2.58	2.58	2.58
GHH	2.20	5.24	6.65	5.24	5.24	5.24
	$\rho = -7$, $\sigma_h = 0.125$					
Separable	0.81	1.24	1.80	1.24	1.24	1.24
$\alpha = 0.5$	1.22	2.49	3.59	2.49	2.49	2.49
GHH	2.20	5.11	8.06	5.11	5.11	5.11
	Sticky-wage model of Auclert et al. (2023)					
Separable	0.93	1.18	1.46	0.94	1.18	1.59

Note: Different values of parameter α correspond to different degrees of complementarity between consumption and hours in preferences, with $\alpha = 0.0$ for separable preferences and $\alpha = 1.0$ for GHH preferences. Different values of the parameter ρ correspond to different degrees of hours complementarities in production.

Figure 4: Response of hours across asset groups under the three-year nominal interest rate peg



Note: The vertical axis records the change in group-specific average weekly hours worked on impact in response to a persistent 1% increase in government spending with the persistence parameter $\rho_G = 0.9$. Model hours are converted into weekly hours as in Appendix C.12. Different values of parameter ρ correspond to different degrees of hours complementarities in production. The lower the ρ, the more complementary hours worked are in production.

6. Sticky-Wage HANK Model with Hours Complementarities

In this section, I present three facts about sticky-wage HANK models. First, I show that a sticky-wage HANK model with heterogeneous hours, separable preferences, and a linear labor aggregation does not solve the HANK trilemma. Second, I introduce hours complementarities in unions' labor aggregation and show that perfect complementarity generates uniform hours across workers, an assumption of the canonical sticky-wage solution. Third, I show that the model with empirically grounded complementarities closes half to three-quarters of the gap between the baseline MPE and the data, almost the same improvement as in the corresponding sticky-price model.

I describe the core model structure but delegate the full description to Appendix E. The household problem is the same as in the sticky-price model of Section 3, except households do not choose hours. Every worker belongs to a continuum of monopolistically competitive unions that choose hours worked. Each union chooses hours worked for every household individually and the union-level wage index to maximize its members' average welfare. Competitive labor packers purchase tasks produced by the unions and sell packaged labor inputs to the intermediate-goods producers. Prices are flexible, and all producers produce the same output using the same amount of labor. The monetary, asset-pricing, and fiscal blocks are the same as in the sticky-price model,

and the model is calibrated to the same set of data targets.

6.1. Unions and the New Keynesian Wage Phillips Curve

Unions aggregate labor using the CES technology familiar from the sticky-price model,

$$N_{kt} = \left(\int_{h \in \mathcal{H}} \int_{x \in \mathcal{X}} x h^{\rho} \mu(h,x)\, dh\, dx \right)^{\frac{1}{\rho}} \left(\int_{h \in \mathcal{H}} \int_{x \in \mathcal{X}} x \mu(h,x)\, dh\, dx \right)^{1-\frac{1}{\rho}},$$

and the union's labor aggregation problem implies a wage-hours menu familiar from Section 3:

$$w_{kt}(h,x) = \tilde{w}_{kt} x H_{kt} \left(\frac{h^{\rho-1}}{\rho H_{kt}^{\rho}} + \frac{1}{h}\left(1 - \frac{1}{\rho}\right) \right).$$

I assume a symmetric equilibrium where all unions choose the same wages $\tilde{w}_{kt} = \tilde{w}_t$ and reference hours $H_{kt} = H_t$, where

$$H_t = \left(\int x_{it} h_{it}^{\rho}\, di \right)^{\frac{1}{\rho}} \left(\int x_{it}\, di \right)^{-\frac{1}{\rho}}.$$

The union's optimality conditions imply a New Keynesian Wage Phillips Curve that describes the dynamics of nominal wages in the model:[45]

$$\begin{aligned} \log\left(1 + \pi_t^w\right) = \kappa_w N_t \Bigg[& - \int \left(\frac{h_{it}}{H_t} \right)^{1-\rho} \left(U_h(c_{it}, h_{it}) + \xi_{i,t} \right) di \\ & - (1 - \tau_t^w) \tilde{w}_t \int \left[\left(\frac{\epsilon_w - 1}{\epsilon_w} - \rho \right) \Phi_{it} + \rho \right] x_{it} U_c(c_{it}, h_{it})\, di \Bigg] \\ & + \beta \log\left(1 + \pi_{t+1}^w\right), \end{aligned} \tag{39}$$

where $\xi_{i,t}$ is the Lagrange multiplier on the non-negativity constraint on hours, and $\Phi_{i,t}$ is the household labor earnings component that depends on hours worked,

$$\Phi_{it} = \frac{1}{\rho} \left(\frac{h_{it}}{H_t} \right)^{\rho} + 1 - \frac{1}{\rho}.$$

6.2. Determination of Hours Worked

The optimality condition with respect to individual hours accounts for both individual utility margins and the union's demand and supply constraints,

$$\varphi h^{\nu} (1 - \alpha + \alpha U_c) = (\lambda_{it}(1 - \tau_t^w)\tilde{w}_t - \lambda_t^w) x_{it} \left(\frac{h_{it}}{H_t} \right)^{\rho-1},$$

[45] I derive the New Keynesian Wage Phillips Curve in Appendix E.2.

where $\lambda_t^w = \lambda_t^N + \lambda_t^H$ equals the sum of the Lagrange multipliers on the CES demand and reference hours constraints. The multiplier on the CES demand constraint reduces individual hours worked, allowing the union to leverage its market power. The multiplier on the reference hours constraint moves hours in the direction that increases labor earnings of households with a high marginal utility of consumption. Formally, the multiplier depends on the covariance between the marginal utility of consumption and the hours-dependent component of labor earnings:[46]

$$\lambda_t^H = -(1-\tau_t^w)\tilde{w}_t(1-\rho)\,\mathrm{Cov}_x\left(U_c, \Phi\right).$$

I focus on the model with separable preferences, where the covariance between the marginal utility of consumption and the hours-dependent earnings component is positive. A strong wealth effect on labor supply means that high marginal utility of consumption prompts households to work longer hours. The effect is so strong that it dominates the market power effect in the calibrated model with separable preferences for any degree of hours complementarity ρ, ensuring that $\lambda_t^w < 0$.

6.3. Perfect Complementarities between Workers' Hours and the Uniform Hours Assumption

The sticky-wage heterogeneous-hours HANK model with hours complementarities microfounds the uniform hours assumption central to the canonical sticky-wage solution of Auclert et al. (2023). When hours are perfect complements, $\rho \to -\infty$, aggregate labor equals the minimum of all workers' hours. Any dispersion in hours is purely wasteful, and the unions choose $h_{it} = H_t$.

Formally, the result follows from the effect hours complementarities have on the intratemporal condition. Under GHH-Plus preferences, the optimality condition for labor is the following:

$$\varphi h_{it}^{\nu}\left(1-\alpha+\alpha U_c(c_{it}, h_{it})\right) = x_{it}\left(\frac{h_{it}}{H_t}\right)^{\rho-1}\left(U_c(c_{it}, h_{it})(1-\tau_t^w)\tilde{w}_t - \lambda_t^w\right). \tag{40}$$

Therefore, households' optimal hours obey

$$h_{it} = H_t^{\frac{1-\rho}{1-\rho+\nu}}\left(\frac{(1-\tau_t^w)\tilde{w}_t x_{it} U_c(c_{it}, h_{it}) - x_{it}\lambda_t^w}{\varphi\left(1-\alpha+\alpha U_c(c_{it}, h_{it})\right)}\right)^{\frac{1}{1-\rho+\nu}}.$$

The first term converges to H_t, since

$$\lim_{\rho\to-\infty} H_t^{\frac{1-\rho}{1-\rho+\nu}} = \lim_{\rho\to-\infty} H_t^{\frac{1}{1+\frac{\nu}{1-\rho}}} = H_t.$$

[46] I derive the result in Appendix E.3.

The second term converges to unity because the idiosyncratic Frisch elasticity converges to zero. For every household with a given level of assets and productivity, the marginal utility of consumption is strictly bounded away from zero and finite for any ρ, as long as a solution exists. With $\lambda_t^w < 0$, the base of the second term is therefore bounded away from zero and finite. The exponent $1/(1-\rho+\nu)$ goes to zero, so the second term converges to one.[47]

Individual hours thus converge to reference hours,

$$\lim_{\rho \to -\infty} h_{it} = H_t,$$

and the ratio of household to reference hours converges to unity. Thus, perfect complementarity in hours microfounds uniform hours across workers, which the canonical sticky-wage model assumes.[48]

6.4. HANK Trilemma in the Sticky-Wage Model with Hours Complementarities

Since perfect complementarity between hours worked generates uniform hours, as in the canonical sticky-wage model, I ask whether empirically-grounded complementarities between workers' hours can solve the HANK trilemma in the sticky-wage model. I calibrate the model to match the same set of targets as the sticky-price model.[49] I compute the MPEs and fiscal multipliers using the same method as in the sticky-price model.

Two facts presented in Figure 5 and Table 8 stand out. First, in the standard model with a linear labor aggregation, the MPE is 0.241, well above the target. Therefore, sticky wages alone are not sufficient to deliver low MPEs in the sticky-wage model with separable preferences, and the uniform hours assumption is key to the canonical sticky-wage model's ability to solve the HANK trilemma.

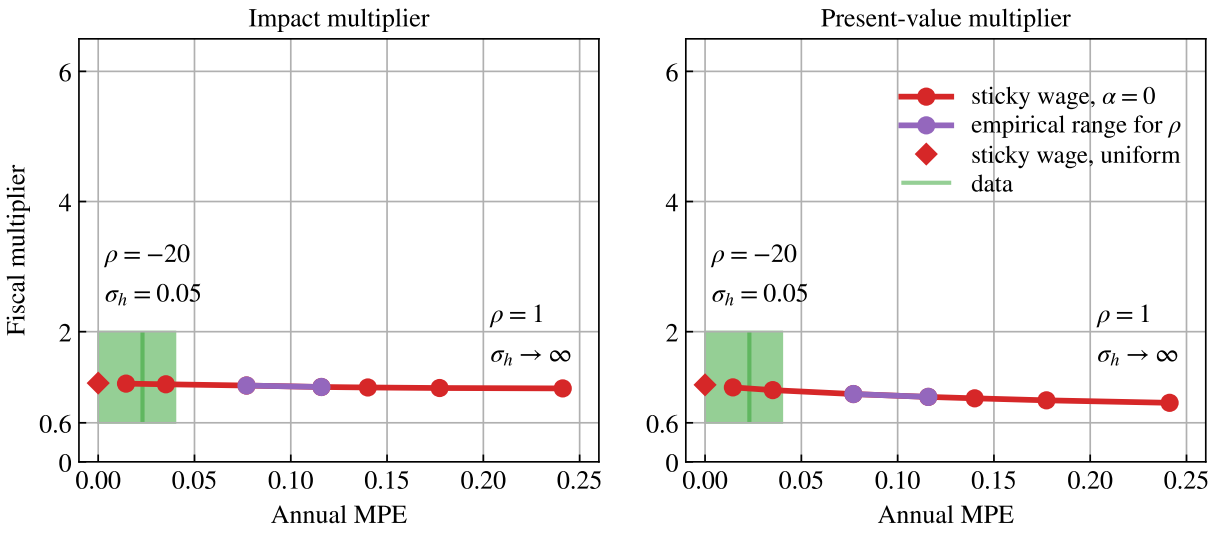
Second, hours complementarities in the range of estimates I used in the sticky-price model yield MPEs in the sticky-wage model that are almost identical to those in the sticky-price model. As Table 8 shows, under the complementarity parameter ρ values of -0.46 and -1.962 as

[47] I verify numerically that the condition $\lambda_t^w < 0$ holds in the sticky-wage model with heterogeneous hours and separable preferences. The condition $\lambda_t^H < 0$ holds because households with high marginal utility of consumption work long hours, and unions push all workers to work longer to increase reference hours and, as a result, increase labor earnings of the high-marginal-utility households.

[48] However, as I show in Appendix E.4, the limit New Keynesian Wage Phillips Curve in the model differs from the NKWPC in the canonical sticky-wage model. The term that reflects the union's interest in how reference hours affect individual workers' earnings does not vanish as $\rho \to -\infty$. Even though hours converge, $h_{it} \to H_t$, productivity becomes more sensitive to dispersion in hours, and the term $(h_{it}/H_t)^\rho$ does not converge to unity.

[49] As in the sticky-price model, since $\int x_{it}\, di = 1$, reference hours equal aggregate hours, $H_t = N_t$.

Figure 5: MPEs and fiscal multipliers in the sticky-wage models with hours complementarities



Note: The left-hand panel plots the impact multiplier, which equals the ratio of the instantaneous output response to the change in government spending. The right-hand panel plots the present-value fiscal multiplier, which equals the ratio of the present values of changes in output and government spending. I specify all models in this plot with separable preferences. Different dots of the same color correspond to different degrees of hours complementarities in labor aggregation by the unions. I highlight the values implied by empirical estimates of hours complementarities from Shao et al. (2023) and Battisti et al. (2024) in purple. The green rectangle is the target range of Auclert et al. (2023) for the MPE and the fiscal multiplier; the vertical green line inside the box is the point estimate of the MPE of 0.023 from Golosov et al. (2024).

Table 8: MPEs and fiscal multipliers in sticky-wage models with hours complementarities

	Separable preferences, $\alpha = 0$						
ρ	1.0	0.5	0.02	-0.46	-1.962	-7.0	-20.0
MPE	0.241	0.177	0.140	0.116	0.077	0.035	0.014
Present-value multiplier	0.907	0.945	0.976	0.998	1.042	1.102	1.144

Note: For each model, I compute annual MPEs from partial-equilibrium transitional dynamics of after-tax labor earnings to an unanticipated transfer shock. The present-value fiscal multiplier equals a ratio of the present values of output and government spending changes, computed from first-order general-equilibrium impulse responses to a persistent government spending shock. I specify all models in this table with separable preferences, $\alpha = 0.0$. Monetary policy follows a constant real rate rule. The parameter ρ controls complementarities between hours worked, with $\rho = 1.0$ corresponding to a standard linear aggregation, where hours are perfect substitutes, and values below zero indicating that hours are gross complements.

in Shao et al. (2023) and Battisti et al. (2024), the MPE is 0.116 and 0.077, respectively. Thus, complementarities close half to three-quarters of the gap between the baseline MPE and the data. However, the empirical evidence comes from studies of firms, not unions, and is not directly portable to the sticky-wage model. Furthermore, while complementarities meaningfully improve the model's fit to the MPE, they do not fully close the gap between the model and the data.[50]

7. Conclusions

Can complementarities between workers' hours solve the HANK trilemma? This paper shows that introducing complementarity between workers' hours in the labor aggregation alleviates the HANK trilemma without fully solving it. The sticky-price and sticky-wage HANK models calibrated to existing empirical estimates of hours complementarities close half to three-quarters of the gap between the baseline MPE and the data. The better fit to the MPE does not come at the expense of the model's fit to fiscal multipliers, which stay between 1.24 and 1.28 in the sticky-price model and between 1.00 and 1.04 in the sticky-wage model, below the empirical upper bound of 1.5. Closing the gap completely requires the elasticity of substitution between hours to be below 0.125, well below the lowest empirically plausible value of 0.338.

[50] In Appendix E.5, I show that when hours are complementary enough to match the MPE, whether prices or wages are sticky has a limited effect on fiscal multipliers across different monetary and fiscal regimes.

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Appendix

A. MPCs, MPEs, and Wage-Hours Dependence

A.1. Regularity Assumptions

Assumption 1. *The per-period utility function $U(c, h)$ is twice continuously differentiable and strictly concave, with $U_c > 0 > U_h$. The after-tax labor earnings function $y(h, \tilde{w}^{\tau,x}, H)$ is twice continuously differentiable, concave, and has strictly positive marginal earnings, $y_h > 0$. Households' optimal choices of consumption and hours are interior.*

Assumption 2. *Aggregate hours H combine individual hours $\{h(a, x)\}_{a\in\mathcal{A},x\in\mathcal{X}}$ through a function that is differentiable, increasing in each household's hours, and homogeneous of degree one.*

A.2. Idiosyncratic Frisch Elasticity of Labor Supply

The idiosyncratic Frisch elasticity is defined in the main text in the following manner:

$$\text{Frisch}_i \equiv \frac{\partial \log h(\lambda, \tilde{w}^{\tau,x}, H)}{\partial \log \tilde{w}^{\tau,x}}.$$

The key assumption underlying the calculation of the idiosyncratic Frisch elasticity is that aggregate hours, H, stay constant in response to the wage variation. Along the Frisch demand curve, consumption and hours adjust to wage shocks in a way that keeps the marginal utility of consumption constant at $\bar{\lambda}$, requiring that

$$\begin{aligned} G(c, h) &= U_c(c, h) - \lambda = 0, \\ \left.\frac{\partial c}{\partial h}\right|_{\lambda=\bar{\lambda}} &= -\frac{G_h}{G_c} = -\frac{U_{ch}}{U_{cc}}. \end{aligned}$$

The optimal intratemporal consumption-leisure condition implicitly defines hours worked h as a function of the after-tax wage factor $\tilde{w}^{\tau,x}$:

$$F(\tilde{w}^{\tau,x}, h) = U_h(c, h) + \lambda \underbrace{\tilde{w}^{\tau,x}\left(f(h, H) + f_h(h, H)h\right)}_{y_h} = 0.$$

Differentiating this condition with respect to $\tilde{w}^{\tau,x}$ and h yields

$$\begin{aligned} F_{\tilde{w}^{\tau,x}} &= \lambda \frac{y_h}{\tilde{w}^{\tau,x}}, \\ F_h &= U_{hc}(c, h)\left.\frac{\partial c}{\partial h}\right|_{\lambda=\bar{\lambda}} + U_{hh}(c, h) + \lambda y_{hh}. \end{aligned} \tag{A.41}$$

Applying the implicit function theorem to $F(\tilde{w}^{\tau,x}, h) = 0$ gives the idiosyncratic Frisch elasticity:

$$\begin{aligned}\text{Frisch}_i &= \left.\frac{\partial \log h}{\partial \log \tilde{w}^{\tau,x}}\right|_{\lambda=\bar{\lambda}} = \left.\frac{\partial h}{\partial \tilde{w}^{\tau,x}}\right|_{\lambda=\bar{\lambda}} \frac{\tilde{w}^{\tau,x}}{h} = -\frac{F_{\tilde{w}^{\tau,x}}}{F_h}\frac{\tilde{w}^{\tau,x}}{h} \\ &= -\frac{\lambda y_h}{U_{hh} - \frac{U_{ch}^2}{U_{cc}} + \lambda y_{hh}}\frac{1}{h} \\ &= -\frac{1}{\frac{U_{hh}h}{\lambda y_h} - \frac{U_{ch}^2 h}{U_{cc}\lambda y_h} + \frac{y_{hh}h}{y_h}}.\end{aligned}$$

Finally, substituting the first-order condition $\lambda = -\frac{U_h}{y_h}$ into this expression yields the desired formula:

$$\text{Frisch}_i = \left(\frac{U_{hh}h}{U_h} - \frac{U_{ch}^2 h}{U_{cc}U_h} - \frac{y_{hh}h}{y_h}\right)^{-1}.$$

How marginal earnings change in hours depends on two effects, and neither of these effects is present in the standard model without wage-hours dependence:

$$\begin{aligned}y_{hh}(h, \tilde{w}^{\tau,x}, H) &= (1-\tau^w)\left(\underbrace{w_h(h,x,H)}_{\text{direct}} + \underbrace{w_{hh}(h,x,H)h + w_h(h,x,H)}_{\text{indirect}}\right) \\ &= (1-\tau^w)\left(w_{hh}(h,x,H)h + 2w_h(h,x,H)\right). \qquad \text{(A.42)}\end{aligned}$$

The direct effect is that the marginal hour is paid at a different wage, with the wage changing at rate $w_h(h,x,H)$. The indirect effect is that the hourly wage changes for every hour already worked, $w_h(h,x,H)h$, and this term moves for two reasons. First, the wage schedule bends, which changes the indirect effect at rate $w_{hh}(h,x,H)h$. Second, there is one more hour to revalue, which changes the indirect effect at rate $w_h(h,x,H)$. Whether earnings display decreasing returns depends not only on whether the hourly wage increases or decreases with hours, but also on the curvature of the wage schedule.

There are two ways changes in hours affect the marginal disutility from working extra time. The first effect is direct and depends on how quickly the marginal disutility grows with hours: $U_{hh}h/U_h$. The second effect is indirect and depends on how consumption affects the marginal disutility from working. Increases in wages increase consumption, and complementarity between consumption and hours makes a positive comovement between the two desirable. Under the regularity Assumption 1, the complementarity between consumption and hours U_{ch} therefore reduces the Frisch elasticity.

A.3. Proof of Lemma 1

Lemma 1. *Under Assumption 2, changes in log aggregate hours* $\log H$ *equal the weighted average of changes in log individual hours* $\log h$*,*

$$d\log H = \int\int \omega(a,x)\, d\log h(a,x)\, da\, dx, \tag{A.43}$$
$$\text{where } \omega(a,x) \equiv \frac{\partial H}{\partial h(a,x)}\frac{h(a,x)}{H} \geq 0 \text{ and } \int\int \omega(a,x)\, da\, dx = 1.$$

Proof. The aggregator is differentiable, so

$$dH = \int\int \frac{\partial H}{\partial h(a,x)}\, dh(a,x)\, da\, dx.$$

Dividing by H and writing each term as $\frac{\partial H}{\partial h(a,x)}\frac{h(a,x)}{H}\frac{dh(a,x)}{h(a,x)}$ gives (A.43).

Homogeneity of degree one means $H(\{s\, h(a,x)\}) = s\, H(\{h(a,x)\})$ for all $s > 0$. Differentiating both sides with respect to s and evaluating at $s = 1$ yields Euler's identity

$$\int\int \frac{\partial H}{\partial h(a,x)}\, h(a,x)\, da\, dx = H. \tag{A.44}$$

Dividing by H gives $\int \omega(a,x)\, da\, dx = 1$. Non-negativity follows from the aggregator being increasing in each household's hours, together with $h(a,x) > 0$ and $H > 0$.

A.4. Proof of Lemma 2

Lemma 2. *Under Assumption 1, household hours respond to perturbations of the after-tax wage factor $\tilde{w}^{\tau,x}$, aggregate hours H, and the marginal utility of consumption $\lambda(a,x)$, according to*

$$\begin{aligned} d\log h(a,x) = &\,\text{Frisch}_i(a,x)\left(d\log\tilde{w}^{\tau,x} + \epsilon_{y_h,H}(a,x)\,d\log H\right) \\ &+ (1-\text{CI}(a,x))\,\text{Frisch}_i(a,x)\,d\log\lambda(a,x), \end{aligned} \tag{A.45}$$

where Frisch_i *is given by* (6)*, and CI is the complementarity index defined in* (9)*.*

Proof. Differentiating the first-order condition with respect to consumption (4) yields:

$$d\lambda = U_{cc}\,dc + U_{ch}\,dh. \tag{A.46}$$

Differentiating the first-order condition with respect to hours (5) yields:

$$(U_{hh} + \lambda y_{hh})\,dh + U_{hc}\,dc + \lambda\frac{y_h}{\tilde{w}^{\tau,x}}\,d\tilde{w}^{\tau,x} + \lambda y_{hH}\,dH + y_h d\lambda = 0,$$

where y_{hh} and y_{hH} each hold the other argument fixed. Substituting for dc from equation (A.46) yields,

$$\underbrace{\left(U_{hh} + \lambda y_{hh} - \frac{U_{ch}^2}{U_{cc}}\right)}_{F_h}\,dh + \lambda\frac{y_h}{\tilde{w}^{\tau,x}}\,d\tilde{w}^{\tau,x} + \lambda y_{hH}\,dH + \left(y_h + \frac{U_{ch}}{U_{cc}}\right)d\lambda = 0, \tag{A.47}$$

Assumption 1 guarantees that F_h is strictly negative and therefore invertible. Since $\lambda = -U_h/y_h$ from (5), $\epsilon_{y_h,H} = y_{hH}H/y_h$, and $\lambda = U_c$,

$$d\log h = -\frac{\lambda y_h}{F_h h}\left(d\log\tilde{w}^{\tau,x} + \epsilon_{y_h,H}\,d\log H + \left(1 - \frac{U_{ch}U_c}{U_h U_{cc}}\right)d\log\lambda\right).$$

Dividing the numerator and denominator of the first multiplicative term on the right-hand side by λy_h and substituting $\lambda = -U_h/y_h$ gives $-\lambda y_h/(F_h h) = -F_{\tilde{w}^{\tau,x}}/F_h \cdot \tilde{w}^{\tau,x}/h = \text{Frisch}_i$, where the last equality comes from Appendix A.2 and equation (A.41). Furthermore, $\text{CI} = U_{ch}U_c/U_{cc}/U_h$ by Lemma 3. The substitutions yield the desired result:

$$d\log h(a,x) = \text{Frisch}_i(a,x)\left(d\log\tilde{w}^{\tau,x} + \epsilon_{y_h,H}(a,x)\,d\log H + (1-\text{CI}(a,x))\,d\log\lambda(a,x)\right).$$

A.5. Proof of Proposition 1

Proposition 1 follows directly from Lemma 1 and 2. Lemma 1 defines how aggregate hours vary along with individual households' hours:

$$d\log H = \int\int \omega(a,x)\, d\log h(a,x)\, da\, dx.$$

Lemma 2 defines how individual households' hours vary with the after-tax wage factor and aggregate hours, and I keep the marginal utility $\lambda(a,x)$ constant:

$$d\log h = \text{Frisch}_i \left(d\log \tilde{w}^{\tau,x} + \epsilon_{y_h,H} d\log H \right).$$

Substituting for the variation in an individual household's hours in the formula for the variation in aggregate hours yields the following.

$$d\log H = \int\int \text{Frisch}_i(a,x) \left(d\log \tilde{w}^{\tau,x} + \epsilon_{y_h,H} d\log H \right) \omega(a,x)\, da\, dx$$

Since the experiment varies the common wage component $\tilde{w}^{\tau}$, I set $d\log x = 0$, so that $d\log \tilde{w}^{\tau,x} = d\log \tilde{w}^{\tau}$. The log changes in the after-tax wage index, $d\log \tilde{w}^{\tau}$, and aggregate hours, $d\log H$, both do not depend on the individual states (a,x). Thus, rearranging terms yields

$$d\log H = \frac{\overline{\text{Frisch}_i}}{1-\Gamma} d\log \tilde{w}^{\tau},$$

where $\overline{\text{Frisch}_i}$ equals the weighted average of household's idiosyncratic Frisch elasticities

$$\overline{\text{Frisch}_i} \equiv \int\int \text{Frisch}_i(a,x)\, \omega(a,x)\, da\, dx,$$

and Γ denotes the weighted average strength of the feedback from aggregate to individual household's hours through marginal earnings

$$\Gamma \equiv \int\int \text{Frisch}_i(a,x)\, \epsilon_{y_h,H}\, \omega(a,x)\, da\, dx,$$

which yields the desired result:

$$\text{Frisch}_a = \frac{d\log H}{d\log \tilde{w}^{\tau}} = \frac{\overline{\text{Frisch}_i}}{1-\Gamma}. \tag{A.48}$$

The aggregate Frisch elasticity is well-defined only if $|\Gamma| < 1$, the condition in the proposition.

To study individual households' responses to aggregate wage shocks, allowing aggregate hours to move, revisit equation (A.45):

$$d\log h = \text{Frisch}_i \left(d\log \tilde{w}^{\tau,x} + \epsilon_{y_h,H}\, d\log H \right).$$

Substituting for the deviation in aggregate hours from equation (A.48),

$$d\log h = \text{Frisch}_i \left(1 + \epsilon_{y_h,H}\text{Frisch}_a\right) d\log \tilde{w}^\tau,$$

yields the desired result

$$\left.\frac{d\log h(a,x)}{d\log \tilde{w}^\tau}\right|_{\{\lambda\}=\bar{\lambda}} = \text{Frisch}_i(a,x)\left(1 + \epsilon_{y_h,H}(a,x)\,\text{Frisch}_a\right)$$

When the elasticity of marginal earnings to aggregate hours is constant across households, the feedback effect Γ simplifies to:

$$\Gamma = \epsilon_{y_h,H}\int\int \text{Frisch}_i(a,x)\,\omega(a,x)\,da\,dx = \epsilon_{y_h,H}\overline{\text{Frisch}_i}.$$

Substituting back into equation (A.48) yields:

$$\frac{1}{\overline{\text{Frisch}_i}} - \frac{1}{\text{Frisch}_a} = \epsilon_{y_h,H}.$$

Allowing the marginal utility of consumption $\lambda(a,x)$ to vary, the same steps yield

$$\begin{aligned} d\log H =& \,\text{Frisch}_a\, d\log \tilde{w}^\tau \\ &+ \int\int (1 - \text{CI}(a,x))\,\frac{\text{Frisch}_i(a,x)}{1-\Gamma}\, d\log\lambda(a,x)\,\omega(a,x)\,da\,dx. \end{aligned}$$

If the idiosyncratic Frisch elasticity is constant across agents, $\text{Frisch}_i/(1-\Gamma) = \text{Frisch}_a$, and the equation reduces to equation (18).

A.6. Proof of Lemma 3

Lemma 3. *Under Assumption 1, the complementarity index, defined as the marginal propensity to consume out of marginal utility-compensated changes in labor earnings induced by a small change in hours worked,*

$$\text{CI} \equiv \frac{\partial c(\lambda, \tilde{w}^{\tau,x}, H)}{\partial \tilde{w}^{\tau,x}} \Bigg/ \underbrace{\tilde{w}^{\tau,x}\left(f(h,H) + f_h(h,H)h\right)}_{y_h} \frac{\partial h(\lambda, \tilde{w}^{\tau,x}, H)}{\partial \tilde{w}^{\tau,x}},$$

can be equivalently expressed in terms of the first and second derivatives of the utility function:

$$\text{CI} = \frac{U_{ch}}{U_{cc}} \frac{U_c}{U_h}. \tag{A.49}$$

To derive the complementarity index, I study the Frisch system, following the approach developed by Auclert et al. (2023) and Azinovic-Yang et al. (2026). I allow for the simultaneous presence of complementarity between consumption and hours and a non-linear mapping from hours to labor earnings.

I derive the Frisch demand system by totally differentiating the static first-order conditions of the household problem

$$U_c = \lambda$$
$$U_h + \lambda y_h = 0,$$

which gives the following:

$$U_{cc}dc + U_{ch}dh = d\lambda$$
$$U_{ch}dc + U_{hh}dh + y_h d\lambda + \lambda \frac{y_h}{\tilde{w}^{\tau,x}} d\tilde{w}^{\tau,x} + \lambda y_{hh}dh + \lambda y_{hH}dH = 0.$$

Dividing the first equation by U_c and the second equation by $U_h = -\lambda y_h$, and using the static optimality relations yields

$$\frac{U_{cc}c}{U_c} d\log c + \frac{U_{ch}h}{U_c} d\log h = d\log\lambda$$
$$\frac{U_{ch}c}{U_h} d\log c + \frac{U_{hh}h}{U_h} d\log h - \frac{y_{hh}h}{y_h} d\log h = d\log\lambda + d\log\tilde{w}^{\tau,x} + \underbrace{\frac{y_{hH}H}{y_h}}_{\epsilon_{y_h,H}} d\log H.$$

Since c and h are the only endogenous variables, the system of equations can be rewritten in the following manner:

$$\underbrace{\begin{bmatrix} \frac{U_{cc}c}{U_c} & \frac{U_{ch}h}{U_c} \\ \frac{U_{ch}c}{U_h} & \frac{U_{hh}h}{U_h} - \frac{y_{hh}h}{y_h} \end{bmatrix}}_{A} \begin{bmatrix} d\log c \\ d\log h \end{bmatrix} = \underbrace{\begin{bmatrix} d\log\lambda \\ d\log\lambda + d\log\tilde{w}^{\tau,x} + \epsilon_{y_h,H}\, d\log H \end{bmatrix}}_{b} \tag{A.50}$$

The determinant of the coefficient matrix, $D = \det(A)$, is the following

$$\begin{aligned} D = \det(A) &= \frac{U_{cc}c}{U_c}\left(\frac{U_{hh}h}{U_h} - \frac{y_{hh}h}{y_h}\right) - \frac{U_{ch}^2 ch}{U_c U_h} \\ &= \frac{ch}{U_c U_h}\left[U_{cc}\left(U_{hh} - U_h\frac{y_{hh}}{y_h}\right) - U_{ch}^2\right] < 0 \end{aligned}$$

The determinant is strictly negative because, by Assumption 1, the utility function is strictly concave, such that $U_{cc} < 0$ and $U_{cc}U_{hh} - U_{ch}^2 > 0$, and the earnings function is strictly increasing and weakly concave such that $U_{cc}U_h y_{hh}/y_h \leq 0$ (combining with $U_c > 0$ and $U_h < 0$ yields the desired inequality). Therefore, the linear system can be inverted to obtain a closed-form solution for Frisch demand elasticities:

$$\begin{bmatrix} d\log c \\ d\log h \end{bmatrix} = \frac{1}{D}\begin{bmatrix} \frac{U_{hh}h}{U_h} - \frac{y_{hh}h}{y_h} & -\frac{U_{ch}h}{U_c} \\ -\frac{U_{ch}c}{U_h} & \frac{U_{cc}c}{U_c} \end{bmatrix}\begin{bmatrix} d\log\lambda \\ d\log\lambda + d\log\tilde{w}^{\tau,x} + \epsilon_{y_h,H}\, d\log H \end{bmatrix} \tag{A.51}$$

I use the result to calculate the complementarity index. From the linear system, it follows that

$$\begin{aligned} \frac{\partial\log c}{\partial\log\tilde{w}^{\tau,x}} &= -\frac{1}{D}\frac{U_{ch}h}{U_c} \\ \frac{\partial\log h}{\partial\log\tilde{w}^{\tau,x}} &= \frac{1}{D}\frac{U_{cc}c}{U_c} \end{aligned}$$

Therefore, the ratio of the derivatives satisfies

$$\frac{\frac{\partial\log c}{\partial\log\tilde{w}^{\tau,x}}}{\frac{\partial\log h}{\partial\log\tilde{w}^{\tau,x}}} = \frac{\frac{\partial c(\lambda,\tilde{w}^{\tau,x},H)}{\partial\tilde{w}^{\tau,x}}\frac{\tilde{w}^{\tau,x}}{c}}{\frac{\partial h(\lambda,\tilde{w}^{\tau,x},H)}{\partial\tilde{w}^{\tau,x}}\frac{\tilde{w}^{\tau,x}}{h}} = \frac{\frac{\partial c(\lambda,\tilde{w}^{\tau,x},H)}{\partial\tilde{w}^{\tau,x}}}{\frac{\partial h(\lambda,\tilde{w}^{\tau,x},H)}{\partial\tilde{w}^{\tau,x}}}\frac{h}{c} = -\frac{U_{ch}}{U_{cc}}\frac{h}{c} \implies \frac{\frac{\partial c(\lambda,\tilde{w}^{\tau,x},H)}{\partial\tilde{w}^{\tau,x}}}{\frac{\partial h(\lambda,\tilde{w}^{\tau,x},H)}{\partial\tilde{w}^{\tau,x}}} = -\frac{U_{ch}}{U_{cc}}$$

Using the definition of the complementarity index

$$\text{CI} \equiv \frac{\partial c(\lambda,\tilde{w}^{\tau,x},H)}{\partial\tilde{w}^{\tau,x}}\Bigg/\underbrace{\tilde{w}^{\tau,x}\left(f(h,H) + f_h(h,H)h\right)}_{y_h}\frac{\partial h(\lambda,\tilde{w}^{\tau,x},H)}{\partial\tilde{w}^{\tau,x}},$$

and substituting for marginal earnings from the intratemporal consumption-hours condition,

$$U_c\, y_h(h,\tilde{w}^{\tau,x},H) = -U_h,$$

yields the desired formula for the complementarity index:

$$\text{CI} = \frac{U_{ch}}{U_{cc}}\frac{U_c}{U_h},$$

which is unchanged from the baseline of Auclert et al. (2023).

A.7. Proof of Proposition 2

The proof closely follows Auclert et al. (2023) and builds on the derivations used to prove Lemma 3. Using the linear system (A.51), calculate the derivative terms:

$$\frac{\partial \log h(\lambda, \tilde{w}^{\tau,x}, H)}{\partial \log \lambda} = \frac{1}{D}\left(\frac{cU_{cc}}{U_c} - \frac{cU_{ch}}{U_h}\right)$$
$$\frac{\partial \log h(\lambda, \tilde{w}^{\tau,x}, H)}{\partial \log \tilde{w}^{\tau,x}} = \frac{1}{D}\frac{cU_{cc}}{U_c} = \text{Frisch}_i$$

Therefore:

$$\frac{\frac{\partial \log h(\lambda,\tilde{w}^{\tau,x},H)}{\partial \log \lambda}}{\frac{\partial \log h(\lambda,\tilde{w}^{\tau,x},H)}{\partial \log \tilde{w}^{\tau,x}}} = \frac{\frac{cU_{cc}}{U_c} - \frac{cU_{ch}}{U_h}}{\frac{cU_{cc}}{U_c}} = 1 - \frac{U_{ch}U_c}{U_{cc}U_h} = 1 - \text{CI} \tag{A.52}$$

Because the elasticity of intertemporal substitution is defined with a negative sign, $\text{EIS} = -\partial \log c(\lambda, \tilde{w}^{\tau,x}, H)/\partial \log \lambda$, the ratio of labor supply and consumption elasticities satisfies:

$$\frac{\frac{\partial \log h(\lambda,\tilde{w}^{\tau,x},H)}{\partial \log \lambda}}{\frac{\partial \log c(\lambda,\tilde{w}^{\tau,x},H)}{\partial \log \lambda}} = \frac{\frac{\partial \log h(\lambda,\tilde{w}^{\tau,x},H)}{\partial \log \lambda}}{\frac{\partial \log h(\lambda,\tilde{w}^{\tau,x},H)}{\partial \log \tilde{w}^{\tau,x}}} \frac{\frac{\partial \log h(\lambda,\tilde{w}^{\tau,x},H)}{\partial \log \tilde{w}^{\tau,x}}}{\frac{\partial \log c(\lambda,\tilde{w}^{\tau,x},H)}{\partial \log \lambda}} = -(1 - \text{CI})\frac{\text{Frisch}_i}{\text{EIS}} \tag{A.53}$$

Then, using the definitions of the marginal propensities to consume and earn:

$$\text{MPC} \equiv \frac{\partial c(a, x, T)}{\partial T}$$

$$\text{MPE} \equiv -\tilde{w}^{\tau,x}\left(f(h, H) + f_h(h, H)h\right)\frac{\partial h(a, x, T)}{\partial T}$$

Therefore, it holds that:

$$\begin{aligned}\frac{\text{MPE}}{\text{MPC}} &= -\frac{\tilde{w}^{\tau,x}\left(f(h, H) + f_h(h, H)h\right)\frac{\partial h(a,x,T)}{\partial T}}{\frac{\partial c(a,x,T)}{\partial T}} \\ &= -\frac{\tilde{w}^{\tau,x}\left(f(h, H) + f_h(h, H)h\right)\frac{\partial \log h(a,x,T)}{\partial \log \lambda}\frac{\partial \log \lambda}{\partial T}h}{\frac{\partial \log c(a,x,T)}{\partial \log \lambda}\frac{\partial \log \lambda}{\partial T}c}\end{aligned}$$

Substituting equation (A.53), whose leading negative sign cancels the one above, and simplifying yields the equation to be proved

$$\frac{\text{MPE}}{\text{MPC}} = \frac{\tilde{w}^{\tau,x} f(h, H)h\left(1 + \frac{f_h(h,H)h}{f(h,H)}\right)}{c}\frac{\text{Frisch}_i}{\text{EIS}}(1 - \text{CI}) \tag{A.54}$$

Denoting after-tax labor income $y(h, \tilde{w}^{\tau,x}, H) = \tilde{w}^{\tau,x} f(h, H)h$ and the elasticity of wages to hours as $\epsilon_{w,h} = f_h(h, H)h/f(h, H)$ yields the formula in the main text.

A.8. Discussion of Proposition 2

The MPE-MPC ratio increases with the elasticity of the hourly wage rate to hours worked, $\epsilon_{w,h}$. The numerator of the first term in Proposition 2 is marginal earnings times hours:

$$y(h, \tilde{w}^{\tau,x}, H)\left(1 + \epsilon_{w,h}\right) = y_h(h, \tilde{w}^{\tau,x}, H)h.$$

When labor earnings are linear in hours, the product of marginal earnings and hours equals total earnings, so the corresponding term in Auclert et al. (2023) is simply the earnings-to-consumption ratio. With wage-hours dependence, this ratio is scaled by a wedge equal to the elasticity of the hourly wage rate to hours. The more the hourly wage rate rises as hours fall, the lower the responsiveness of labor earnings to changes in hours, and the lower the MPE.

The MPE-MPC ratio decreases with the complementarity index that regulates the strength of the wealth effect on labor supply. A preference for more positive comovement between consumption and hours reduces the wealth effect on labor supply and the MPE.

The MPE-MPC ratio decreases with the elasticity of intertemporal substitution. The higher the EIS, the more consumption rather than hours moves with the marginal utility of consumption, and the lower the MPE-MPC ratio.

A.9. Proof of Proposition 3

This section proves Proposition 3. Since the production function is linear in labor,

$$Y = zH - F,$$

government spending only affects output by affecting aggregate hours H:

$$dY = z\, dH = zH\, d\log H.$$

As Section 2 demonstrated, individual hours depend on the after-tax wage factor $\tilde{w}^{\tau,x}$, the Lagrange multiplier λ, and on aggregate hours H. Equation (18) leverages the fact that aggregate hours are homogeneous of degree one in individual hours, to decompose changes in aggregate hours into the wage and wealth channels, which already absorb the effect of aggregate hours:

$$\begin{aligned} d\log H = &\,\text{Frisch}_a\, d\log \tilde{w}^{\tau} \\ &+ \text{Frisch}_a \int\int (1 - \text{CI}(a,x))\, d\log \lambda(a,x)\, \omega(a,x)\, da\, dx, \end{aligned}$$

where the wealth effect is potentially heterogeneous across households. Government spending moves the after-tax wage index and the marginal utility of consumption, $d\log \tilde{w}^{\tau} = \Omega_{\tilde{w}^{\tau},G}\, d\log G$ and $d\log \lambda(a,x) = \Omega_{\lambda,G}(a,x)\, d\log G$. Expressing the change in output in terms of elasticities yields:

$$\begin{aligned} dY = &\, z\,\frac{H}{G}\frac{\partial \log H}{\partial \log \tilde{w}^{\tau}}\frac{\partial \log \tilde{w}^{\tau}}{\partial \log G}\, dG \\ &+ z\,\frac{H}{G}\left[\int\int \frac{\partial \log h(a,x)}{\partial \log \lambda(a,x)}\frac{\partial \log \lambda(a,x)}{\partial \log G}\frac{\partial H}{\partial h(a,x)}\frac{h(a,x)}{H}\, da\, dx\right] dG. \end{aligned}$$

Next, using notation familiar from Proposition 1, $\omega(a,x) = \partial H/\partial h(a,x) \cdot h(a,x)/H$, and substituting for the partial derivatives of log aggregate hours with respect to $\log \tilde{w}^{\tau}$ and $\log \lambda(a,x)$:

$$dY = \frac{zH}{G}\text{Frisch}_a\left[\Omega_{\tilde{w}^{\tau},G} + \int\int (1 - \text{CI}(a,x))\, \Omega_{\lambda,G}(a,x)\, \omega(a,x)\, da\, dx\right] dG,$$

where $\Omega_{\tilde{w}^{\tau},G} = \partial \log \tilde{w}^{\tau}/\partial \log G$ and $\Omega_{\lambda,G}(a,x) = \partial \log \lambda(a,x)/\partial \log G$ denote the general-equilibrium wage and wealth effects of government spending shocks.

A.10. Proof of Proposition A1

Proposition A1 follows from solving the Frisch system for the relationship between consumption and the Lagrange multiplier on the budget constraint and expressing it in terms of the idiosyncratic Frisch elasticity and the complementarity index between consumption and hours—objects defined in Section 2 in the main text.

Start from the linear system (A.51), derived in the proof of Lemma 3, which already relates $d\log c$ to $d\log\lambda$. The elasticity of intertemporal substitution varies the marginal utility of consumption holding the after-tax wage factor and aggregate hours fixed, so set $d\log\tilde{w}^{\tau,x} = d\log H = 0$ and read the first row of the system:

$$\text{EIS} = -\frac{\partial\log c}{\partial\log\lambda} = -\frac{1}{D}\left(\frac{U_{hh}h}{U_h} - \frac{y_{hh}h}{y_h} - \frac{U_{ch}h}{U_c}\right).$$

The determinant is given by the following formula

$$D = \frac{U_{cc}c}{U_c}\left(\frac{U_{hh}h}{U_h} - \frac{y_{hh}h}{y_h}\right) - \frac{U_{ch}^2 ch}{U_c U_h}.$$

Factoring $U_{cc}c/U_c$ out of the determinant D yields

$$\text{EIS} = -\frac{\partial\log c}{\partial\log\lambda} = -\frac{U_c}{U_{cc}c}\left(\frac{\frac{U_{hh}h}{U_h} - \frac{y_{hh}h}{y_h} - \frac{U_{ch}h}{U_c}}{\frac{U_{hh}h}{U_h} - \frac{y_{hh}h}{y_h} - \frac{U_{ch}^2}{U_{cc}}\frac{h}{U_h}}\right).$$

Next, use the formulas for the idiosyncratic Frisch elasticity and the complementarity index

$$\text{Frisch}_i = \left(\frac{U_{hh}h}{U_h} - \frac{U_{ch}^2 h}{U_{cc}U_h} - \frac{y_{hh}h}{y_h}\right)^{-1}$$
$$\text{CI} = \frac{U_{ch}U_c}{U_{cc}U_h}$$

to derive the following expression for the EIS

$$\begin{aligned}\text{EIS} = -\frac{\partial\log c}{\partial\log\lambda} &= -\frac{U_c}{U_{cc}c}\left(1 + \text{Frisch}_i\left(\frac{U_{ch}^2}{U_{cc}}\frac{h}{U_h} - \frac{U_{ch}h}{U_c}\right)\right)\\ &= -\frac{U_c}{U_{cc}c}\left(1 + \text{Frisch}_i\left(\text{CI}\frac{U_{ch}h}{U_c} - \frac{U_{ch}h}{U_c}\right)\right)\\ &= -\frac{U_c}{U_{cc}c}\left(1 - \text{Frisch}_i\,(1-\text{CI})\,\frac{U_{ch}h}{U_c}\right).\end{aligned}$$

B. Labor Supply Elasticities

B.1. Aggregate Frisch Elasticity of Labor Supply

In this section, I prove that the aggregate Frisch elasticity of labor supply does not depend on the degree of hours complementarity in production. Computing the aggregate Frisch elasticity corresponds to a thought experiment that varies the wage rate that all workers face, allows reference hours H to adjust endogenously, but keeps the marginal utility of consumption λ constant. Keeping the marginal utility of consumption constant means that the thought experiment shuts down any potential changes in the wealth distribution or real interest rate, which would only affect labor supply through the shadow price on the budget constraint.

The idea of the proof is to aggregate individual households' labor supply choices into reference hours and solve for the total hours adjustment by substituting back into the households' optimality condition. This procedure corresponds to analytically computing a fixed point between household and reference hours, holding the distribution of households and their marginal utility of consumption constant. Combining the optimal consumption-leisure choice (32) with GHH-Plus preferences implies the following optimal choice of hours worked:

$$h_{it}^{*} = \left((1-\tau_t^w)\tilde{w}_t x_{it}\frac{\lambda_{it}}{\varphi\left(\alpha\lambda_{it}+1-\alpha\right)}\right)^{\frac{1}{1-\rho+\nu}} H_t^{\frac{1-\rho}{1-\rho+\nu}}.$$

Next, I aggregate individual hours using the CES aggregator from the main text, dropping individual subscripts inside the aggregator.

$$\begin{aligned}
H_t &= \mathbb{E}_x\left[h^{\rho}\right]^{\frac{1}{\rho}} \\
&= \mathbb{E}_x\left[\left((1-\tau_t^w)\tilde{w}_t x\frac{\lambda}{\varphi\left(\alpha\lambda+1-\alpha\right)}\right)^{\frac{\rho}{1-\rho+\nu}}\left(H_t^{\frac{1-\rho}{1-\rho+\nu}}\right)^{\rho}\right]^{\frac{1}{\rho}} \\
&= H_t^{\frac{1-\rho}{1-\rho+\nu}}\left((1-\tau_t^w)\tilde{w}_t\right)^{\frac{1}{1-\rho+\nu}}\mathbb{E}_x\left[\left(\frac{x}{\varphi}\frac{\lambda}{\alpha\lambda+1-\alpha}\right)^{\frac{\rho}{1-\rho+\nu}}\right]^{\frac{1}{\rho}}
\end{aligned}$$

Rearranging gives an expression for reference hours

$$H_t = \left((1-\tau_t^w)\tilde{w}_t\right)^{\frac{1}{\nu}}\mathbb{E}_x\left[\left(\frac{x}{\varphi}\frac{\lambda}{\alpha\lambda+1-\alpha}\right)^{\frac{\rho}{1-\rho+\nu}}\right]^{\frac{1-\rho+\nu}{\nu\rho}}. \tag{B.55}$$

The result follows from applying the definition of the aggregate Frisch elasticity to equation (B.55):

$$\text{Frisch}_a \equiv \left.\frac{\partial \log H_t}{\partial \log \tilde{w}_t^{\tau}}\right|_{\{\lambda_{it}\}=\bar{\lambda}_i} = \frac{1}{\nu}.$$

Furthermore, substituting back into the optimal choice of individual hours yields

$$h^* = \left((1-\tau_t^w)\tilde{w}_t\right)^{\frac{1}{\nu}} \left(\frac{x}{\varphi}\frac{\lambda}{\alpha\lambda+1-\alpha}\right)^{\frac{1}{1-\rho+\nu}} \mathbb{E}_x\left[\left(\frac{x}{\varphi}\frac{\lambda}{\alpha\lambda+1-\alpha}\right)^{\frac{\rho}{1-\rho+\nu}}\right]^{\frac{1-\rho}{\nu\rho}}. \tag{B.56}$$

Therefore, the Frisch elasticity of individual hours to the after-tax wage index that allows aggregate hours to vary equals the aggregate Frisch elasticity

$$\left.\frac{\partial \log h_{it}}{\partial \log \tilde{w}_t^{\tau}}\right|_{\{\lambda_{it}\}=\bar{\lambda}_i} = \frac{1}{\nu}.$$

C. Computational Appendix

C.1. Steady State of the Household Block

The solution uses the endogenous grid method of Carroll (2006) and is based on Auclert et al. (2021) and Auclert et al. (2023). The algorithm is adjusted to solve the household problem subject to the equilibrium earnings function derived from the firm's problem with imperfect substitutability between hours in production.

I set up a grid for the future level of asset $a' \in \mathcal{G}^a$ and approximate the expected derivative of the value function tomorrow, $W(a', x) \equiv \mathbb{E}\left[V_a(a', x') \mid x\right]$, starting from a guess that the household consumes a constant fraction of its cash on hand. I use the Euler equation $W(a', x) = \lambda' = U_c(a', x)$ to recover $c(a(a'), x)$ and $h(a(a'), x)$ from the static first-order conditions of the problem. That is, combining the first-order condition w.r.t. c, a', and the envelope condition (equations (25), (27), and (28)) yields:

$$\beta(1+r)W(a', x) = \beta(1+r)\mathbb{E}\left[V_a(a', x')|x\right] = \left(c - \varphi\alpha\frac{h^{1+\nu}}{1+\nu}\right)^{-\sigma} = U_c(a', x) \tag{C.57}$$

Using the fact that we know $\beta(1+r)W(a', x) = U_c(a', x)$, one can recover the optimal labor supply from (26):

$$h(a(a'), x) = \left(\frac{U_c(a', x)\tilde{w}^{\tau, x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}-1}}{\varphi\left(\alpha U_c(a', x) + (1-\alpha)\right)}\right)^{\frac{1}{1-\rho+\nu}} \tag{C.58}$$

Then, from the first-order condition with respect to consumption (25) and using the just-calculated optimal labor supply, one can recover the optimal consumption:

$$c(a(a'), x) = U_c(a', x)^{-\frac{1}{\sigma}} + \varphi\alpha\frac{h(a', x)^{1+\nu}}{1+\nu} \tag{C.59}$$

The recovered policy functions c and h are defined on the endogenous grid for current assets $a(a') \notin \mathcal{G}^a$. I use linear interpolation to recover policy functions defined on the exogenous asset grid—$c(a, x)$ and $h(a, x)$—such that $a \in \mathcal{G}^a$. Linear interpolation uses the fact that writing the budget constraint in the following fashion:

$$a'(a', x) + c(a', x) - \tilde{w}^{\tau, x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}}\left(\frac{h(a', x)^\rho}{\rho\mathbb{E}[h^\rho]} + 1 - \frac{1}{\rho}\right) - T = (1+r)a(a') \tag{C.60}$$

This equation implicitly defines an endogenous grid for current assets $a(a')$, which is different from the exogenous grid $\mathcal{G}^a$. The iteration procedure requires that one updates $V_a(a)$ and that current assets a are on the exogenous grid $a \in \mathcal{G}^a$. Therefore, using linear interpolation, we look for current assets that lie on the exogenous grid $\mathcal{G}^a$, and, at the same time, obtain the future

level of assets $a'(a)$ now defined on the exogenous grid $a \in \mathcal{G}^a$. Once assets today are on the exogenous grid $a \in \mathcal{G}^a$, we can recover assets tomorrow $a'(a,x)$ from the budget constraint:

$$a'(a,x) = \tilde{w}^{\tau,x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}}\left(\frac{h(a,x)^\rho}{\rho\mathbb{E}[h^\rho]} + 1 - \frac{1}{\rho}\right) + T + (1+r)a - c(a,x) \tag{C.61}$$

Once we find the correct policy functions for the constrained agents as described in the subsection below, the iteration step closes with an update of the derivative of the value function using the envelope theorem $V_a(a) = (1+r)\lambda = (1+r)U_c$ using the policy function $c(a,x)$ and $h(a,x)$ where $a \in \mathcal{G}^a$.

C.2. The Constrained Households

Some agents would like to choose $a'(a,x) < 0$, which would violate the borrowing constraint $a'(a,x) > 0$. For those agents, I set $a'(a,x)|_{bc} = 0$ and solve for their policy functions $c(a,x)|_{bc}$, $h(a,x)|_{bc}$ which maximize the following objective function

$$\begin{aligned}
&\max_{c,h} \frac{1}{1-\sigma}\left(c - \varphi\alpha\frac{h^{1+\nu}}{1+\nu}\right)^{1-\sigma} - \varphi(1-\alpha)\frac{h^{1+\nu}}{1+\nu} \\
&\text{s.t.} \quad c = \tilde{w}^{\tau,x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}}\left(\frac{h^\rho}{\rho\mathbb{E}[h^\rho]} + 1 - \frac{1}{\rho}\right) + T + (1+r)a
\end{aligned}$$

The first-order conditions of the static constrained optimization problem:

$$\begin{aligned}
\left(c - \varphi\alpha\frac{h^{1+\nu}}{1+\nu}\right)^{-\sigma} &= \lambda \\
\varphi h^\nu\left(\alpha\left(c - \varphi\alpha\frac{h^{1+\nu}}{1+\nu}\right)^{-\sigma} + (1-\alpha)\right) &= \lambda\tilde{w}^{\tau,x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}-1}h^{\rho-1}
\end{aligned}$$

The problem is a system of 2 equations in 2 unknowns:

$$\begin{aligned}
\varphi h^\nu\left(\alpha\left(c - \varphi\alpha\frac{h^{1+\nu}}{1+\nu}\right)^{-\sigma} + (1-\alpha)\right) &= \left(c - \varphi\alpha\frac{h^{1+\nu}}{1+\nu}\right)^{-\sigma}\tilde{w}^{\tau,x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}-1}h^{\rho-1} \\
\tilde{w}^{\tau,x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}}\left(\frac{h^\rho}{\rho\mathbb{E}[h^\rho]} + 1 - \frac{1}{\rho}\right) + T + (1+r)a &= c
\end{aligned}$$

I solve this system of equations using the algorithm of Auclert et al. (2023) because of its computational efficiency. The algorithm needs to be adjusted to handle the non-linear earnings properly; I provide the full algorithm description below.

The algorithm solves the budget constraint using Newton's updates incorporating the first-order conditions. The total derivative of the expenditure function takes into account the first-order conditions. To make the function more linear, use the logarithm of marginal utility $\log U_c$ as an argument of the expenditure function.

The general structure of Newton's method is as follows: Approximate a function $f(x)$ to first order at a point x^*

$$f(x) \approx f(x^*) + f'(x^*)(x - x^*)$$

Then, in approximation, it holds that:

$$x \approx x^* + \frac{f(x) - f(x^*)}{f'(x^*)}$$

We want the target point x to be the root of the function f such that $f(x) = 0$, then:

$$x \approx x^* - \frac{f(x^*)}{f'(x^*)}$$

The approximation motivates the iteration procedure in Newton's method.

$$x^{(n+1)} = x^{(n)} - \frac{f(x^n)}{f'(x^n)}$$

The function f is the budget constraint, and the argument $x = \log U_c$ is equal to the logarithm of the marginal utility. The budget constraint is evaluated using the policy functions $c = c(\log U_c)$ and $h = h(\log U_c)$, which are both functions of $\log U_c$.

$$f(\log U_c) = c - \tilde{w}^{\tau,x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}}\left(\frac{h^\rho}{\rho\mathbb{E}[h^\rho]} + 1 - \frac{1}{\rho}\right) - T$$

and the derivative of the expenditure function with respect to $\log U_c$

$$f'(\log U_c) = \frac{\partial f}{\partial \log U_c} = \frac{\partial c}{\partial \log U_c} - \tilde{w}^{\tau,x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}-1}h^{\rho-1}\frac{\partial h}{\partial \log U_c}$$

It follows that Newton's iteration, in this case, is the following

$$\log U_c^{(n+1)} = \log U_c^{(n)} - \frac{c - \tilde{w}^{\tau,x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}}\left(\frac{h^\rho}{\rho\mathbb{E}[h^\rho]} + 1 - \frac{1}{\rho}\right) - T}{\frac{\partial c(\log U_c)}{\partial \log U_c} - \tilde{w}^{\tau,x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}-1}h^{\rho-1}\frac{\partial h(\log U_c)}{\partial \log U_c}} \tag{C.62}$$

Taking the first-order condition w.r.t. labor and substituting out $\lambda = U_c$, one obtains

$$\varphi h^\nu\left(\alpha U_c + 1 - \alpha\right) = U_c\tilde{w}^{\tau,x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}-1}h^{\rho-1}$$

Taking logs, I get an implicit function

$$F(\log h, \log U_c) = (1 - \rho + \nu)\log h - \log\left(\tilde{w}^{\tau,x}\mathbb{E}[h^\rho]^{\frac{1}{\rho}-1}\right) + \log\varphi - \log U_c + \log\left(1 - \alpha + \alpha U_c\right)$$

Applying the implicit function theorem to recover $\frac{\partial h}{\partial \log U_c}$, one obtains:

$$F_{\log h} = \frac{\partial F}{\partial \log h} = 1 - \rho + \nu.$$

$$F_{\log U_c} = \frac{\partial F}{\partial \log U_c} = -1 + \frac{\partial \log\left(1 - \alpha + \alpha U_c\right)}{\partial U_c} U_c = -1 + \frac{\alpha U_c}{1 - \alpha + \alpha U_c} = -\frac{1 - \alpha}{1 - \alpha + \alpha U_c}$$

$$\frac{\partial \log h}{\partial \log U_c} = -\frac{F_{\log U_c}}{F_{\log h}} = \frac{1}{1 - \rho + \nu} \frac{1 - \alpha}{1 - \alpha + \alpha U_c}$$

$$\frac{\partial h}{\partial \log U_c} = h \frac{\partial \log h}{\partial \log U_c} = \frac{h}{1 - \rho + \nu} \frac{1 - \alpha}{1 - \alpha + \alpha U_c} \tag{C.63}$$

The derivative $\frac{\partial c}{\partial \log U_c}$ can be recovered from the marginal utility function.

$$U_c = \left(c - \varphi\alpha \frac{h^{1+\nu}}{1+\nu}\right)^{-\sigma}$$

Taking logs, one obtains the following implicit function

$$F(\log U_c, c) = \log U_c + \sigma \log\left(c - \varphi\alpha \frac{h^{1+\nu}}{1+\nu}\right)$$

Using the same approach as before

$$F_{\log U_c} = \frac{\partial F}{\partial \log U_c} = 1 - \frac{\sigma}{c - \varphi\alpha \frac{h^{1+\nu}}{1+\nu}} \varphi\alpha h^{\nu} \frac{\partial h}{\partial \log U_c}$$

$$F_c = \frac{\partial F}{\partial c} = \frac{\sigma}{c - \varphi\alpha \frac{h^{1+\nu}}{1+\nu}}$$

$$\frac{\partial c}{\partial \log U_c} = -\frac{F_{\log U_c}}{F_c} = \varphi\alpha h^{\nu} \frac{\partial h}{\partial \log U_c} - \frac{1}{\sigma}\left(c - \varphi\alpha \frac{h^{1+\nu}}{1+\nu}\right) = \varphi\alpha h^{\nu} \frac{\partial h}{\partial \log U_c} - \frac{1}{\sigma} U_c^{-\frac{1}{\sigma}} \tag{C.64}$$

After providing an initial guess for the marginal utility, using, for example, the GHH utility case, all that remains is to bring the iteration (C.62) to convergence using the partial derivatives (C.63) and (C.64).

C.3. Elasticity of Intertemporal Substitution with GHH Plus Preferences

Note that the formula (C.64) provides a direct way to compute the elasticity of intertemporal substitution for the GHH Plus preferences.

$$\text{EIS} = -\frac{\partial \log c(\lambda, \tilde{w}^{\tau,x}, H)}{\partial \log \lambda} = -\frac{1}{c} \frac{\partial c}{\partial \log U_c}$$

The following proposition establishes how wage-hours dependence affects the elasticity of intertemporal substitution.[51]

[51] I provide the proof of Proposition A1 in Appendix A.10.

Proposition A1. *For any individual in the state* (a, x)

$$\text{EIS} = -\frac{U_c}{U_{cc}c}\left(1 - \text{Frisch}_i\,(1 - \text{CI})\,\frac{U_{ch}h}{U_c}\right). \tag{C.65}$$

Consistent with Proposition A1, wage-hours dependence introduced through hours complementarities has no direct effect on the EIS when preferences are separable (CI = 0) or GHH (CI = 1). In the first case, hours do not affect the marginal utility of consumption directly. In the second case, changes in the marginal utility of consumption have no effect on hours worked. However, in the intermediate range where consumption and hours are complementary but with a complementarity index of less than one, wage-hours dependence affects the EIS by moderating how the wealth effect on labor supply affects hours and consumption. With decreasing returns to hours, $y_{hh} < 0$, the idiosyncratic Frisch elasticity declines, reducing the optimal movements in hours worked. Since the wealth effect moves consumption and hours in opposite directions and both affect the marginal utility of consumption, a smaller responsiveness of the hours margin leads to a larger responsiveness of the consumption margin, and a higher EIS. Table C.4 shows that the calibrated curvature parameter σ does not vary with ρ in the separable cases, where the EIS is exactly $1/\sigma$. The differences in the GHH cases are present because, for each household, the EIS depends on the level of consumption and hours, which are affected by complementarities between hours worked in production. In the intermediate cases, both effects are present and work in opposite directions, which is why σ rises with complementarities at low α and falls at high α.

C.4. Properties of the Production Function

Economic interpretation. The production function is linear in the labor input and includes a fixed cost. The labor input is a productivity-weighted CES aggregator,

$$n_{jt} = \left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\mu^j(h,x)h^{\frac{\sigma_h-1}{\sigma_h}}dhdx\right)^{\frac{\sigma_h}{\sigma_h-1}}\left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\mu^j(h,x)dhdx\right)^{-\frac{1}{\sigma_h-1}}, \tag{C.66}$$

where $\mu^j(h,x)$ is the measure of workers supplying h hours with idiosyncratic productivity x, and σ_h denotes the elasticity of substitution between individual hours worked. This formulation captures the idea that time spent at work is more productive when co-workers are simultaneously present. The labor input can be equivalently decomposed into the product of the total headcount, average idiosyncratic productivity, and effective hours per worker:

$$n_{jt} = \underbrace{\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} d\mu^j(h,x)}_{\text{Headcount}} \cdot \underbrace{\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} \frac{x\,d\mu^j(h,x)}{\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} d\mu^j(h,x)}}_{\text{Average idiosyncratic productivity}} \cdot \underbrace{H_{jt}}_{\text{Effective hours per worker}}, \tag{C.67}$$

The first term in equation (C.67) is a scaling factor. Given a constant productivity-weighted composition of hours worked, an increase in employment leads to a proportional increase in production. The second term represents the productivity composition of the workforce employed at the firm. The third term is effective hours per worker, or reference hours H_{jt}. Mathematically, reference hours are the *power mean of hours worked*.

$$H_{jt} = \mathbb{E}_x \left[h^\rho\right]^{\frac{1}{\rho}} = \left(\int_{x\in\mathcal{X}} \int_{h\in\mathcal{H}} h^\rho \underbrace{\frac{x\mu^j(h,x)}{\int_{x\in\mathcal{X}} \int_{h\in\mathcal{H}} x\mu^j(h,x)dhdx}}_{\text{Density over}(h,x)} dhdx \right)^{\frac{1}{\rho}}$$

For $\rho = (\sigma_h - 1)/\sigma_h < 1$, dispersion in hours lowers production.

Constant returns to scale. The second term in equation (C.66) ensures constant returns to labor. If employment for all types increases λ times, the production increases λ times.

$$\lambda y_{jt} = z \left(\int_{x\in\mathcal{X}} \int_{h\in\mathcal{H}} x\left(\lambda\mu^j(h,x)\right) h^\rho dhdx \right)^{\frac{1}{\rho}} \left(\int_{x\in\mathcal{X}} \int_{h\in\mathcal{H}} x\left(\lambda\mu^j(h,x)\right) dhdx \right)^{1-\frac{1}{\rho}}$$

Likewise, if all workers decide to work λ times more hours, production increases λ times.

$$\lambda y_{jt} = z \left(\int_{x\in\mathcal{X}} \int_{h\in\mathcal{H}} x\mu^j(h,x)\left(\lambda h^\rho\right) dhdx \right)^{\frac{1}{\rho}} \left(\int_{x\in\mathcal{X}} \int_{h\in\mathcal{H}} x\mu^j(h,x)dhdx \right)^{1-\frac{1}{\rho}}$$

In other words, there are constant returns to the *number of employees* as well as to *hours worked* of all employees. In particular, without the second term for $\rho \in (0,1)$, there would be increasing returns to scale. For $\rho < 0$, there would be negative returns to scale.

Constant elasticity of substitution between hours worked. The production function y_{jt} features constant elasticity of substitution between hours worked. Without loss of generality, consider two inputs h_1 and h_2. The elasticity of substitution measures the change in optimal factor shares (in %) in response to a 1% change in the marginal rate of technical substitution between the factors.

$$\sigma_h = -\frac{d\ln\left(\frac{h_1}{h_2}\right)}{d\ln\left(\frac{q_{h_1}}{q_{h_2}}\right)}$$

q_{h_1} and q_{h_2} denote marginal products of hours worked.

$$q_{h_1} = \frac{dy_{jt}}{dh_1} = z\mathbb{E}[h^\rho]^{\frac{1-\rho}{\rho}} h_1^{\rho-1} \int_{x\in\mathcal{X}} x\mu^j(h,x)dx$$
$$q_{h_2} = \frac{dy_{jt}}{dh_2} = z\mathbb{E}[h^\rho]^{\frac{1-\rho}{\rho}} h_2^{\rho-1} \int_{x\in\mathcal{X}} x\mu^j(h,x)dx$$

It follows that the marginal rate of technical substitution is proportional to the input ratio.

$$\frac{q_{h_1}}{q_{h_2}} = \left(\frac{h_1}{h_2}\right)^{\rho-1} \frac{\int_{x\in\mathcal{X}} x\mu^j(h,x)dx}{\int_{x\in\mathcal{X}} x\mu^j(h,x)}$$

Calculate the derivative of the MRTS with respect to a change in the input ratio.

$$\frac{d\left(\frac{q_{h_1}}{q_{h_2}}\right)}{d\left(\frac{h_1}{h_2}\right)} = (\rho-1)\left(\frac{h_1}{h_2}\right)^{\rho-2} \frac{\int_{x\in\mathcal{X}} x\mu^j(h,x)dx}{\int_{x\in\mathcal{X}} x\mu^j(h,x)}$$

Putting everything together yields the following formula for the elasticity of substitution between different hours worked.

$$\sigma_h = -\frac{d\ln\left(\frac{h_1}{h_2}\right)}{d\ln\left(\frac{q_{h_1}}{q_{h_2}}\right)} = -\frac{d\left(\frac{h_1}{h_2}\right)}{d\left(\frac{q_{h_1}}{q_{h_2}}\right)}\frac{\frac{q_{h_1}}{q_{h_2}}}{\frac{h_1}{h_2}} = \frac{1}{1-\rho}$$

The inverse of $1-\rho$ regulates the elasticity of substitution between hours worked.

Dispersion in hours worked decreases production. Start with a production technology where everybody works the same hours $\bar{h}$ and has equal productivity $\bar{x} = 1$. There is a unit measure of workers employed $\mu^j(\bar{h},\bar{x}) = 1$. Denote the produced output as $\bar{q}$.

$$\bar{q} = z\bar{x}\bar{h}$$

Consider a mean-preserving spread of hours worked in the firm. A measure 0.5 works $\bar{h}-\epsilon$, and a measure 0.5 works $\bar{h}+\epsilon$ hours.

$$q_{new} = z\left(\frac{1}{2}\left(\bar{h}-\epsilon\right)^{\rho} + \frac{1}{2}\left(\bar{h}+\epsilon\right)^{\rho}\right)^{\frac{1}{\rho}}$$

By Jensen's inequality, $q_{new} < \bar{q}$ for any $\rho < 1$. Jensen's inequality also applies to the production function without the simplifying assumptions. For two firms with an identical number and productivity of workers but different production technologies with $\rho_1 < \rho_2$, the second firm is less productive.

$$z\mathbb{E}_x[h^{\rho_1}]^{\frac{1}{\rho_1}}\left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\mu^j(h,x)dhdx\right) < z\mathbb{E}_x[h^{\rho_2}]^{\frac{1}{\rho_2}}\left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\mu^j(h,x)dhdx\right)$$

In other words, dispersion in hours worked is more costly for firms with stronger hours complementarities.

C.5. Intermediate-Goods Producer's Problem

Intermediate-goods producers choose output y_{jt}, labor input n_{jt}, workforce composition $\{\mu^j(h,x)\}_{h\in\mathcal{H},x\in\mathcal{X}}$, and price p_{jt} to maximize the present discounted value of the firm. I solve the model under certainty equivalence. Therefore, no arbitrage dictates that equity and bonds should yield the real rate r_t^e, which is the appropriate discount rate for the firm problem. Price setting is subject to quadratic price adjustment cost $\frac{\psi}{2}\log\left(\frac{p_{jt}}{p_{j,t-1}}\right)^2 Y_t$, as in Rotemberg (1982), where ψ regulates the size of the adjustment cost.

$$
\begin{aligned}
J_t(p_{j,t-1}) = &\max_{y_{jt},n_{jt},\{\mu^j(h,x)\},p_{jt}} d_{jt} + \frac{1}{1+r_t^e}J_{t+1}(p_{jt}) \\
\text{s.t. } d_{jt} =&\frac{p_{jt}}{P_t}y_{jt} - \int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} w(h,x)h\mu^j(h,x)dhdx - \frac{\psi}{2}\log\left(\frac{p_{jt}}{p_{j,t-1}}\right)^2 Y_t \\
y_{jt} =&\left(\frac{p_{jt}}{P_t}\right)^{-\varepsilon_p} Y_t \\
y_{jt} =&zn_{jt} - f \\
n_{jt} =&\left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\mu^j(h,x)h^{\frac{\sigma_h-1}{\sigma_h}}dhdx\right)^{\frac{\sigma_h}{\sigma_h-1}} \\
&\cdot\left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\mu^j(h,x)dhdx\right)^{-\frac{1}{\sigma_h-1}}
\end{aligned}
$$

Substitute the definition of dividends into the objective function and write down the stage Lagrangian.

$$
\begin{aligned}
\mathcal{L}\left(p_{jt},y_{jt},n_{jt},\{\mu^j(h,x)\};\lambda_{1t},\lambda_{2t},\lambda_{3t}\right) = &\frac{p_{jt}}{P_t}y_{jt} - \int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} w(h,x)h\mu^j(h,x)dhdx \\
&- \frac{\psi}{2}\left[\log\left(\frac{p_{jt}}{p_{j,t-1}}\right)\right]^2 Y_t + \frac{1}{1+r_t^e}J_{t+1}(p_{jt}) \\
&+ \lambda_{1t}\left(\left(\frac{p_{jt}}{P_t}\right)^{-\varepsilon_p} Y_t - y_{jt}\right) \\
&+ \lambda_{2t}\left(zn_{jt} - f - y_{jt}\right) \\
&+ \lambda_{3t}\left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\mu^j(h,x)h^{\frac{\sigma_h-1}{\sigma_h}}dhdx\right)^{\frac{\sigma_h}{\sigma_h-1}} \\
&\cdot\left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\mu^j(h,x)dhdx\right)^{-\frac{1}{\sigma_h-1}} \\
&- \lambda_{3t}n_{jt}
\end{aligned}
$$

The first-order conditions are the following.

$$
\frac{\partial\mathcal{L}}{\partial p_{jt}}: \frac{y_{jt}}{P_t} - \psi\log\left(\frac{p_{jt}}{p_{j,t-1}}\right)\frac{Y_t}{p_{jt}} + \frac{1}{1+r_t^e}J_{p,t+1}(p_{jt}) - \lambda_{1t}\varepsilon_p\left(\frac{p_{jt}}{P_t}\right)^{-\varepsilon_p-1}\frac{Y_t}{P_t} = 0
$$

$$\frac{\partial \mathcal{L}}{\partial y_{jt}}: \frac{p_{jt}}{P_t} - \lambda_{1t} - \lambda_{2t} = 0$$
$$\frac{\partial \mathcal{L}}{\partial n_{jt}}: \lambda_{2t} z - \lambda_{3t} = 0$$
$$\frac{\partial \mathcal{L}}{\partial \mu^j(h,x)}: -w(h,x)h + \lambda_{3t}\frac{x h^\rho}{\rho}\mathbb{E}_x\left[h^\rho\right]^{\frac{1-\rho}{\rho}} + \lambda_{3t} x\left(1-\frac{1}{\rho}\right)\mathbb{E}_x\left[h^\rho\right]^{\frac{1}{\rho}} = 0$$

Combining the last two first-order conditions and noticing that the Lagrange multiplier on the production function is equal to the marginal cost gives the following wage equation

$$w(h,x) = \tilde{w}_t x \mathbb{E}_x\left[h^\rho\right]^{\frac{1}{\rho}}\left(\frac{h^{\rho-1}}{\rho\mathbb{E}_x\left[h^\rho\right]} + \frac{1}{h}\left(1-\frac{1}{\rho}\right)\right),$$

where $\tilde{w}_t = mc_t \cdot z$.

To derive the New Keynesian Phillips Curve, first derive the envelope condition.

$$J_{p,t}(p_{j,t-1}) = \psi \log\left(\frac{p_{jt}}{p_{j,t-1}}\right)\frac{Y_t}{p_{j,t-1}} \tag{C.68}$$

Next, the Lagrange multiplier on the CES demand constraint can be expressed as a function of the marginal cost $mc_t = \lambda_{2t}$.

$$\lambda_{1t} = \frac{p_{jt}}{P_t} - \lambda_{2t} = \frac{p_{jt}}{P_t} - mc_t \tag{C.69}$$

Substituting the derivative of the value function from the envelope condition (C.68) and the CES demand Lagrange multiplier from equation (C.69) yields the following.

$$\psi \log\left(\frac{p_{jt}}{p_{j,t-1}}\right)\frac{Y_t}{p_{jt}} = \frac{y_{jt}}{P_t} + \frac{1}{1+r_t^e}\psi \log\left(\frac{p_{j,t+1}}{p_{jt}}\right)\frac{Y_{t+1}}{p_{jt}} - \left(\frac{p_{jt}}{P_t} - mc_t\right)\varepsilon_p\left(\frac{p_{jt}}{P_t}\right)^{-\varepsilon_p-1}\frac{Y_t}{P_t}$$

Rearranging terms and using the definition of inflation $\pi_t = \frac{p_{jt}-p_{j,t-1}}{p_{j,t-1}}$ yields the New Keynesian Phillips Curve.

$$\log(1+\pi_t) = \frac{1-\varepsilon_p}{\psi}\left(\frac{p_{jt}}{P_t}\right)^{1-\varepsilon_p} + \frac{\varepsilon_p}{\psi}\left(\frac{p_{jt}}{P_t}\right)^{-\varepsilon_p} mc_t + \frac{1}{1+r_t^e}\frac{Y_{t+1}}{Y_t}\log(1+\pi_{t+1})$$

I assume a symmetric equilibrium where all firms set the same price p_{jt}, which gives the New Keynesian Phillips Curve from the main text.

$$\log(1+\pi_t) = \kappa_p\left(mc_t - \frac{\varepsilon_p - 1}{\varepsilon_p}\right) + \frac{1}{1+r_t^e}\frac{Y_{t+1}}{Y_t}\log(1+\pi_{t+1})$$

C.6. Steady-State MPC

The model is quarterly, and the steady-state calibration targets MPC = 0.25, broadly consistent with empirical evidence in Johnson et al. (2006) and Fagereng et al. (2021), among

many others. As in Auclert et al. (2023), I use discount factor heterogeneity to target aggregate savings and MPCs consistent with empirical evidence. A sufficiently high discount factor ensures that households generate savings consistent with the data. A sufficiently low discount factor yields the steady-state MPC of 0.25.

In the steady state, I calculate the MPC and the MPE using the finite differences method to approximate the derivatives. In particular, I calculate the MPC and the MPE for each household characterized by a pair of states (a_i, x) in the following fashion:

$$\text{MPC}(a,x) = \frac{c(a_{i+1},x) - c(a_{i-1},x)}{(1+r)\,(a_{i+1}-a_{i-1})}$$
$$\text{MPE}(a,x) = \frac{y(a_{i+1},x) - y(a_{i-1},x)}{(1+r)\,(a_{i+1}-a_{i-1})}$$

For individuals at the maximum gridpoint, n_a, I take a finite difference between point n_a and $n_a - 1$.

For households that are borrowing-constrained, back out the MPE from an analytical relationship derived in equation (17) in the main text.

$$\frac{\text{MPE}(\underline{a},x)}{\text{MPC}(\underline{a},x)} = \frac{y(\underline{a},x)(1+\epsilon_{w,h}(\underline{a},x))}{c(\underline{a},x)} \frac{\text{Frisch}}{\text{EIS}(\underline{a},x)} \left(1 - \text{CI}(\underline{a},x)\right)$$

The formula yields a ratio of the MPE and MPC. Combining the formula above with the differentiated budget constraint: MPC+MPE = 1 yields the MPC and MPE for each borrowing-constrained agent. Most variables needed to calculate the MPE-MPC ratio are available from the solution of the household problem.

The labor earnings and consumption for the borrowing-constrained household are readily available from the solution of the household problem. The Frisch elasticity to idiosyncratic shocks equals $\frac{1}{1-\rho+\nu}$. One can calculate the elasticity of intertemporal substitution from the formula (C.64).

$$\text{EIS} = -\frac{\partial \log c}{\partial \log U_c} = -\frac{1}{c}\left(\varphi\alpha h^{\nu}\frac{\partial h}{\partial \log U_c} - \frac{1}{\sigma}U_c^{-\frac{1}{\sigma}}\right)$$

The complementarity index is defined in the main text as $\text{CI} = \frac{\alpha U_c}{1-\alpha+\alpha U_c}$. The remaining object that I need to derive is the wage elasticity to hours worked $\epsilon_{w,h} = \frac{f_h(h,H)h}{f(h,H)}$. $f(h,H)$ is the wage-hours dependence function introduced in (2), which in the quantitative model takes the following form.

$$f(h,H) = H\left(\frac{h^{\rho-1}}{\rho H^{\rho}} + \frac{1}{h}\left(1 - \frac{1}{\rho}\right)\right)$$

Differentiating with respect to hours yields a formula for the elasticity $\epsilon_{w,h}$ of wages to hours worked.

$$f_h(h,H) = H\left(\frac{\rho-1}{\rho}\frac{h^{\rho-2}}{H^{\rho}} - \frac{1}{h^2}\left(1 - \frac{1}{\rho}\right)\right)$$

$$= \left(1 - \frac{1}{\rho}\right) H \frac{1}{h^2} \left(\left(\frac{h}{H}\right)^{\rho} - 1\right)$$

$$\epsilon_{w,h} = \left(1 - \frac{1}{\rho}\right) \frac{\frac{1}{h} H \left(\frac{h^{\rho}}{H^{\rho}} - 1\right)}{H \left(\frac{h^{\rho-1}}{\rho H^{\rho}} + \frac{1}{h}\left(1 - \frac{1}{\rho}\right)\right)} = \left(1 - \frac{1}{\rho}\right) \frac{\left(\left(\frac{h}{H}\right)^{\rho-1} - \frac{H}{h}\right)}{\frac{1}{\rho}\left(\frac{h}{H}\right)^{\rho-1} + \frac{H}{h}\left(1 - \frac{1}{\rho}\right)}$$

$$1 + \epsilon_{w,h} = \frac{\left(\frac{h}{H}\right)^{\rho-1}}{H \left(\frac{h^{\rho-1}}{\rho H^{\rho}} + \frac{1}{h}\left(1 - \frac{1}{\rho}\right)\right)}$$

The last formula for $1 + \epsilon_{w,h}$ corresponds directly to the implementation in the code.

MPE in the Model and in the Data

The model is quarterly, and almost all empirical evidence about the MPEs is available at annual frequencies. To match the model to the data, I follow Auclert et al. (2023) and simulate a transition path of individual household choices in response to a one-time wealth shock. I calculate the model MPE as a mean partial equilibrium cumulative response of labor earnings over the four-quarter horizon to a small transfer shock.

$$\text{MPE} = \sum_{s=0}^{3} \frac{\partial Y_s^{labor}}{\partial T_0}$$

Moreover, it is necessary to determine what MPE values are compatible with empirical evidence. Auclert et al. (2023) argue that the direct MPE measure from lottery winnings in Swedish administrative data in Cesarini et al. (2017) was the best available evidence when their paper was published. Cesarini et al. (2017) find an annual average MPE of 0.01. On the other hand, Imbens et al. (2001) study the impact of 20-year-long annuity payments on labor supply. The MPE in Imbens et al. (2001) measures a change in labor earnings per annual payment from a 20-year annuity. Auclert et al. (2023) note that in the permanent income model with unit discount factor and gross interest rate, receiving an annuity over 20 years is equivalent to receiving the whole amount immediately. Thus, the model MPE equals $1/20$ of the Imbens et al. (2001) MPE. However, the closer an agent is to the borrowing constraint, the more the labor supply response is concentrated in the same period. Thus, the ratio of the two MPE measures is model-dependent and reflects the prevalence of borrowing constraints. In the baseline model of Auclert et al. (2023), the ratio of the Imbens et al. (2001) MPE to the model MPE is 3.6. Combined with the maximum MPE estimate of 0.122 in Imbens et al. (2001), Auclert et al. (2023) consider MPE values between 0 and $0.122/3.6 \approx 0.04$ an acceptable range.

The most recent estimates of the MPE based on high-quality administrative data from the US in Golosov et al. (2024) are within the target range put forward by Auclert et al. (2023). Golosov et al. (2024) find an annual average MPE = 0.023 at a household level. This value is very close to the middle of the original interval in Auclert et al. (2023).

Moreover, I redo the calculation of the MPE ratio for the 20-year annuity and one-time winning in Auclert et al. (2023) for the model with complementarities between hours worked. For separable preferences, hours as gross complements in production with $\rho = -0.46$, the ratio of the Imbens et al. (2001) and Auclert et al. (2023) MPEs is 3.4, little changed from the baseline model. Therefore, I use the original MPE interval from Auclert et al. (2023).

Additional considerations. A prominent feature of the labor earnings responses to unexpected lottery winnings is their high persistence. Very high *MPEs out of changes in unearned income* estimated by Golosov et al. (2024) suggest that the baseline one-year MPE measure does not fully capture the nature of labor supply adjustment. In particular, the labor earnings responses in Cesarini et al. (2017) and Golosov et al. (2024) are moderate compared to the size of the wealth shock. However, the response is persistent and builds up to a substantial fraction of the total lottery size over the lifetime. To further compare the model's performance with the estimates in Golosov et al. (2024), I calculate the lifetime MPE as in Golosov et al. (2024). In particular, I calculate the following measure:

$$\text{MPE}_{Golosov} = -\frac{1}{5}\sum_{y=0}^{4}\frac{\sum_{t=0}^{3}\sum_{x\in\mathcal{X}}\sum_{a\in\mathcal{A}}\left(D_{4y+t}(a,x)y_{4y+t}^{\text{pretax}}(a,x) - D_{ss}(a,x)y_{ss}^{\text{pretax}}(a,x)\right)}{\sum_{t=0}^{3}\left((1+r)A_{4y+t-1} - A_{4y+t} - rA_{ss} + \mathbb{1}_{4y+t=0}dT\right)}$$

where $y_{4y+t}^{\text{pretax}}(a,x)$ denotes pre-tax labor earnings in quarter $q = 4y+t$, A_{4y+t} denotes aggregate savings chosen in quarter $4y+t$, and $A_{-1} = A_{ss}$. In other words, I calculate transitional dynamics to a one-time wealth shock over a five-year horizon, sum quarterly responses within each year, and calculate the average response of labor earnings to the change in an unearned income. I use the fact that the change in unearned income in the initial period is equal to the dT-sized wealth shock. The wealth shock is idiosyncratic, and I keep the interest rate at its steady-state level r.

Lifetime MPE out of unearned income is high in the canonical model with separable preferences and perfect substitutability between hours worked in production, in line with empirical evidence in Golosov et al. (2024) (see Table C.1). Lifetime MPEs decline substantially when the model features complementarities between hours worked in production or nonseparable preferences. These results indicate that it is difficult to match low annual MPEs and high lifetime MPEs out of unearned income. Resolving this tension is beyond the scope of this paper.

Table C.1: Lifetime MPEs out of unearned income across preferences and hours complementarities

	α										
ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.59	0.51	0.45	0.40	0.35	0.29	0.23	0.18	0.12	0.06	0.00
0.5	0.51	0.44	0.39	0.34	0.29	0.24	0.19	0.15	0.10	0.05	0.00
0.02	0.45	0.39	0.34	0.30	0.25	0.21	0.17	0.12	0.08	0.04	0.00
-0.46	0.40	0.35	0.31	0.26	0.22	0.18	0.15	0.11	0.07	0.03	0.00
-1.962	0.30	0.26	0.23	0.19	0.16	0.13	0.10	0.08	0.05	0.02	0.00
-7	0.15	0.14	0.12	0.10	0.08	0.07	0.05	0.04	0.02	0.01	0.00

Note: For each model, I compute the lifetime MPE out of unearned income as in Golosov et al. (2024). The parameter α controls the degree of complementarity between consumption and hours, with $\alpha = 0$ corresponding to separable preferences and $\alpha = 1.0$ corresponding to GHH preferences. The parameter ρ controls complementarities between hours worked, with $\rho = 1.0$ corresponding to a standard linear aggregation, where hours are perfect substitutes, and values below zero indicating that hours are gross complements.

C.7. Aggregation

The labor market encompasses submarkets for worker types (h, x). Therefore, the market clearing condition on each market requires that the demanded measures $\mu(h, x)$ are equal to the measure of household supplying this combination of hours and productivity.

$$\forall_{h,x} \quad \mu(h,x) = \int_{\mathcal{A}} D(a,x) \mathbb{1}\left\{h(a,x) = h\right\} da$$

The model features a unit mass of workers, and both sides of the equation integrate to 1.

$$\int_{x\in\mathcal{X}} \int_{h\in\mathcal{H}} \mu(h,x) dh dx = \int_{x\in\mathcal{X}} \int_{a\in\mathcal{A}} D(a,x) \left(\int_{h\in\mathcal{H}} \mathbb{1}\left\{h(a,x) = h\right\} dh \right) da dx = 1$$

Integration over the entire state space $\mathcal{A}\times\mathcal{X}$ must also yield a unit mass of households. Therefore, integration over hours worked h, and x equals the integration over a and x.

$$\int_{x\in\mathcal{X}} \int_{h\in\mathcal{H}} \mu(h,x) dh dx = \int_{x\in\mathcal{X}} \int_{a\in\mathcal{A}} D(a,x) da dx = 1$$

In the next steps, I provide a discrete approximation to the labor market variables from the main text. First, calculate the expected worker productivity over the measure of workers $\mu(h, x)$.

$$\mathbb{E}[x] \equiv \int_{x\in\mathcal{X}} \int_{h\in\mathcal{H}} x\mu(h,x) dh dx = \int_{x\in\mathcal{X}} \int_{a\in\mathcal{A}} xD(a,x) da dx = 1 \tag{C.70}$$

Calculate an analogous integral for hours h^ρ.

$$\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} xh^\rho\mu(h,x)dhdx = \int_{x\in\mathcal{X}}\int_{a\in\mathcal{A}} xh(a,x)^\rho D(a,x)dadx \tag{C.71}$$

Introduce a notation for the expected efficiency hours.

$$\mathbb{E}_x\left[h^\rho\right] \equiv \int_{x\in\mathcal{X}}\int_{a\in\mathcal{A}} h(a,x)^\rho \frac{xD(a,x)}{\int_{x\in\mathcal{X}}\int_{a\in\mathcal{A}} xD(a,x)dadx}dadx \tag{C.72}$$

Aggregation requires the approximation of the integrals from the main text. I approximate the aggregate distribution on the discrete grid for assets $\mathcal{G}^a$ and for the idiosyncratic income states $\mathcal{G}^x$. The distribution affects the labor market where firms optimized over worker masses $\mu(h,x)$ (and $\mu(h,x) = \mu(h,x)$ due to symmetry). Approximate the integral over the idiosyncratic state x using a discretized grid $\mathcal{G}^x$.

$$\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} xh^\rho\mu(h,x)dhdx \approx \sum_{x\in\mathcal{G}^x}\int_{h\in\mathcal{H}} xh^\rho\mu(h,x)dh \tag{C.73}$$

The total mass of agents is unity regardless of whether we sum over masses of agents working a specific number of hours or having a specific amount of assets.

$$\sum_{x\in\mathcal{G}^x}\int_{h\in\mathcal{H}}\mu(h,x)dh = 1 \approx \sum_{x\in\mathcal{G}^x}\sum_{a\in\mathcal{G}^a} D(a,x) \tag{C.74}$$

Similarly, use that integration over assets and hours are equivalent and approximate to obtain a fraction of agents at the idiosyncratic state x.

$$\int_{h\in\mathcal{H}}\mu(h,x)dh = \int_{a\in\mathcal{A}}\mu(a,x)da \approx \sum_{a\in\mathcal{G}^a} D(a,x) \equiv D(x) \tag{C.75}$$

The expectation of the income state in the model is equal to unity, such that:

$$1 = \mathbb{E}[x] \approx \sum_{x\in\mathcal{G}^x} xD(x) \tag{C.76}$$

Using these results, one can derive aggregate hours N as a function of assets and income states.

$$\begin{aligned}
N &= \left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\mu(h,x)h^\rho dhdx\right)^{\frac{1}{\rho}}\left(\int_{x\in\mathcal{X}}\int_{h\in\mathcal{H}} x\mu(h,x)dhdx\right)^{1-\frac{1}{\rho}} \\
&\approx \left(\sum_{x\in\mathcal{G}^x}\int_{h\in\mathcal{H}} xh^\rho\frac{\mu(h,x)}{\sum_{x\in\mathcal{G}^x}\int_{h\in\mathcal{H}} x\mu(h,x)dh}dh\right)^{\frac{1}{\rho}}\left(\sum_{x\in\mathcal{G}^x}\int_{h\in\mathcal{H}} x\mu(h,x)dh\right) \\
&\overset{(C.73)}{\approx} \left(\sum_{x\in\mathcal{G}^x} x\int_{h\in\mathcal{H}} h^\rho\frac{\mu(h,x)}{\sum_{x\in\mathcal{G}^x} x\sum_{a\in\mathcal{G}^a} D(a,x)}dh\right)^{\frac{1}{\rho}}\left(\sum_{x\in\mathcal{G}^x} x\sum_{a\in\mathcal{G}^a} D(a,x)\right)
\end{aligned}$$

$$\overset{(C.75)}{=} \left(\sum_{x \in \mathcal{G}^x} x \int_{h \in \mathcal{H}} h^\rho \mu(h,x) dh \right)^{\frac{1}{\rho}} \left(\sum_{x \in \mathcal{G}^x} x D(x) \right)$$

$$\overset{(C.76)}{\approx} \left(\sum_{x \in \mathcal{G}^x} x \int_{h \in \mathcal{H}} h^\rho \mu(h,x) dh \right)^{\frac{1}{\rho}}$$

$$\approx \left(\sum_{x \in \mathcal{G}^x} \sum_{a \in \mathcal{G}^a} x h(a,x)^\rho D(a,x) \right)^{\frac{1}{\rho}} \tag{C.77}$$

Using this approximation, one recovers the aggregate labor input N and, by extension, the total production Y. The total firm wage bill is also a function of N. Since all households work, and the average productivity is 1, one also recovers reference hours $H = N$ for free.

C.8. Total Wage Bill

First, using the definition of the wage bill $\int_{x \in \mathcal{X}} \int_{h \in \mathcal{H}} w(h,x) h \mu(h,x) dh dx$ and the optimal wage schedule $w(h,x) = \tilde{w} x \mathbb{E}_x[h^\rho]^{\frac{1}{\rho}} \left[\frac{1}{\rho} \frac{h^{\rho-1}}{\mathbb{E}_x[h^\rho]} + \frac{1}{h} \left(1 - \frac{1}{\rho} \right) \right]$, one derives the following.

$$\int_{x \in \mathcal{X}} \int_{h \in \mathcal{H}} w(h,x) h \mu(h,x) dh dx = \int_{x \in \mathcal{X}} \int_{h \in \mathcal{H}} \tilde{w} x \mathbb{E}_x[h^\rho]^{\frac{1}{\rho}} \left(\frac{h^\rho}{\rho \mathbb{E}_x[h^\rho]} + 1 - \frac{1}{\rho} \right) \mu(h,x) dh dx =$$
$$\tilde{w} \mathbb{E}_x[h^\rho]^{\frac{1}{\rho}} \left(\frac{1}{\rho \mathbb{E}_x[h^\rho]} \int_{x \in \mathcal{X}} \int_{h \in \mathcal{H}} x h^\rho \mu(h,x) dh dx + 1 - \frac{1}{\rho} \right)$$

Using the definition of the expectation term $\mathbb{E}_x[h^\rho] \equiv \int_{x \in \mathcal{X}} \int_{h \in \mathcal{H}} x h^\rho \frac{\mu(h,x)}{\int_{x \in \mathcal{X}} \int_{h \in \mathcal{H}} x \mu(h,x) dh dx} dh dx$ from the main text, and that $\mathbb{E}[x] = \int_{x \in \mathcal{X}} \int_{h \in \mathcal{H}} x \mu(h,x) dh dx = 1$, the total wage bill can be expressed in terms of $\tilde{w}$ and H.

$$\int_{x \in \mathcal{X}} \int_{h \in \mathcal{H}} w(h,x) h \mu(h,x) dh dx = \tilde{w} \mathbb{E}_x[h^\rho]^{\frac{1}{\rho}} = \tilde{w} H$$

The wage bill is therefore proportional to the aggregate wage index $\tilde{w}$ even though the wage schedule $w(h,x)$ is nonlinear in individual hours. This property is used in Appendix D. As established above, $H = N$, so one can calculate the total wage bill using the expression for N from equation (C.77):

$$\tilde{w} N = \tilde{w} H \approx \tilde{w} \left(\sum_{x \in \mathcal{G}^x} \sum_{a \in \mathcal{G}^a} x h(a,x)^\rho D(a,x) \right)^{\frac{1}{\rho}}.$$

C.9. Internally-Calibrated Parameters across Models

Table C.2: Upper discount factor β_u across preferences and hours complementarities

	α										
ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	0.978	0.981	0.983	0.984	0.985	0.985	0.985	0.986	0.986	0.987	0.987
-0.46	0.977	0.980	0.982	0.983	0.984	0.984	0.985	0.986	0.986	0.987	0.987
-1.962	0.978	0.980	0.981	0.982	0.983	0.984	0.984	0.985	0.986	0.986	0.986
-7	0.977	0.979	0.980	0.981	0.982	0.983	0.983	0.984	0.984	0.985	0.985

Note: I calibrate the upper discount factor to match aggregate liquid asset demand of 1.4 times annual GDP. The parameter α controls the degree of complementarity between consumption and hours, with $\alpha = 0$ corresponding to separable preferences and $\alpha = 1.0$ corresponding to GHH preferences. The parameter ρ controls complementarities between hours worked, with $\rho = 1.0$ corresponding to a standard linear aggregation, where hours are perfect substitutes, and values below zero indicating that hours are gross complements.

Table C.3: Lower discount factor β_l across preferences and hours complementarities

	α										
ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	0.923	0.938	0.947	0.952	0.956	0.962	0.965	0.967	0.969	0.970	0.970
-0.46	0.940	0.943	0.946	0.950	0.953	0.956	0.959	0.960	0.962	0.963	0.963
-1.962	0.941	0.944	0.947	0.949	0.952	0.953	0.955	0.956	0.957	0.958	0.958
-7	0.948	0.948	0.948	0.948	0.948	0.948	0.947	0.947	0.947	0.947	0.946

Note: I calibrate the lower discount factor to match the aggregate quarterly MPC of 0.25. The parameter α controls the degree of complementarity between consumption and hours, with $\alpha = 0$ corresponding to separable preferences and $\alpha = 1.0$ corresponding to GHH preferences. The parameter ρ controls complementarities between hours worked, with $\rho = 1.0$ corresponding to a standard linear aggregation, where hours are perfect substitutes, and values below zero indicating that hours are gross complements.

Table C.4: Utility curvature parameter σ across preferences and hours complementarities

	α										
ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	2.000	1.746	1.639	1.573	1.529	1.501	1.487	1.484	1.493	1.513	1.544
-0.46	2.000	1.813	1.711	1.637	1.581	1.538	1.505	1.481	1.465	1.456	1.453
-1.962	2.000	1.843	1.744	1.667	1.604	1.552	1.509	1.474	1.446	1.424	1.408
-7	2.000	1.880	1.785	1.701	1.627	1.561	1.503	1.452	1.408	1.371	1.339

Note: I calibrate the utility curvature parameter σ to match the average EIS of 0.5. The parameter α controls the degree of complementarity between consumption and hours, with $\alpha = 0$ corresponding to separable preferences and $\alpha = 1.0$ corresponding to GHH preferences. The parameter ρ controls complementarities between hours worked, with $\rho = 1.0$ corresponding to a standard linear aggregation, where hours are perfect substitutes, and values below zero indicating that hours are gross complements.

Table C.5: Labor disutility parameter φ across preferences and hours complementarities

	α										
ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	0.784	0.827	0.848	0.862	0.871	0.879	0.882	0.883	0.880	0.874	0.865
-0.46	0.755	0.797	0.821	0.836	0.845	0.847	0.843	0.835	0.823	0.808	0.790
-1.962	0.735	0.778	0.805	0.821	0.829	0.829	0.823	0.812	0.797	0.779	0.758
-7	0.707	0.748	0.777	0.795	0.804	0.802	0.793	0.779	0.760	0.738	0.716

Note: I calibrate the labor disutility parameter φ such that aggregate labor supply equals labor demand $N^d = 1$. The parameter α controls the degree of complementarity between consumption and hours, with $\alpha = 0$ corresponding to separable preferences and $\alpha = 1.0$ corresponding to GHH preferences. The parameter ρ controls complementarities between hours worked, with $\rho = 1.0$ corresponding to a standard linear aggregation, where hours are perfect substitutes, and values below zero indicating that hours are gross complements.

C.10. Variance of Log Idiosyncratic Productivity

This section reports the calibrated value of the log idiosyncratic productivity term $\log x$. For separable preferences $\alpha = 0.0$, the wage penalties incurred by part-time workers more than offset a higher concentration of hours worked and a lower standard deviation of log idiosyncratic income terms $\sigma_{\log x}$ is required to match the empirical variance of log labor earnings.

For nonseparable preferences, the choice of hours depends more on productivity, and

there are relatively more workers working high hours (see Figure C.1). The impact of more concentrated hours prevails, and a higher standard deviation of $\log x$ is required to match the empirical variance of log labor earnings.

I follow Auclert et al. (2023) and do not separately model the transitory and persistent shock components, contrary to Krueger et al. (2016). For comparison, the corresponding estimate of $\sigma_{\log x}$ in Krueger et al. (2016) derives from the following formula.

$$\sigma_{\log x} = \sqrt{\frac{\sigma_\eta^2}{1-\phi^2} + \sigma_\epsilon^2}$$

Krueger et al. (2016) estimate the persistence of the permanent shock component $\phi = 0.9695$, variance of the persistent shock $\sigma_\eta^2 = 0.0384$, and variance of the transitory shock $\sigma_\epsilon^2 = 0.0522$. Substituting into the formula yields $\sigma_{\log x} = 0.8315$.

Table C.6: Standard deviation of log idiosyncratic productivity $\sigma_{\log x}$ across preferences and hours complementarities

	α										
ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	0.95	0.85	0.80	0.76	0.73	0.70	0.68	0.66	0.64	0.63	0.61
0.5	0.95	0.86	0.81	0.77	0.74	0.72	0.70	0.68	0.66	0.64	0.63
0.02	0.94	0.87	0.82	0.79	0.76	0.73	0.71	0.69	0.67	0.66	0.65
-0.46	0.94	0.87	0.83	0.80	0.77	0.74	0.72	0.70	0.69	0.67	0.66
-1.962	0.93	0.89	0.85	0.82	0.80	0.77	0.76	0.74	0.72	0.71	0.70
-7	0.93	0.90	0.88	0.86	0.84	0.83	0.82	0.80	0.79	0.78	0.77

Note: I calibrate the variance of log idiosyncratic productivity to match a cross-sectional standard deviation of log labor earnings of 0.92. The parameter α controls the degree of complementarity between consumption and hours, with $\alpha = 0$ corresponding to separable preferences and $\alpha = 1.0$ corresponding to GHH preferences. The parameter ρ controls complementarities between hours worked, with $\rho = 1.0$ corresponding to a standard linear aggregation, where hours are perfect substitutes, and values below zero indicating that hours are gross complements.

The calibrated standard deviation of log idiosyncratic productivity, $\sigma_{\log x}$, slightly declines with complementarity in hours under separable preferences, but moderately increases with complementarity under nonseparable preferences. This occurs because hours complementarities affect labor earnings dispersion through two competing channels: by compressing hours worked

around the reference level and by introducing wage penalties that inflate wage dispersion when part-time workers outnumber overtime workers (as occurs under separable preferences).

C.11. Negative Labor Earnings in the Model with Hours Complementarities

If reference hours H are positive and $\rho < 1$, labor earnings become negative for sufficiently low hours.

$$\lim_{h\to 0} w(h)h = \tilde{w}x\mathbb{E}\left[h^{\rho}\right]^{\frac{1}{\rho}}\left(\lim_{h\to 0}\frac{h^{\rho}}{\rho\mathbb{E}_x\left[h^{\rho}\right]} + 1 - \frac{1}{\rho}\right) < 0$$

In the model with hours complementarities, choosing $h = 0$ has a severe adverse impact on firm production, and the equilibrium wage is negative. Therefore, in contrast to the standard model with perfectly substitutable hours, $h = 0$ cannot be interpreted as non-participation.

Table C.7 reports the fraction of households with negative labor earnings for each model parametrization. The share of households with negative labor earnings increases with the degree of complementarity between hours worked in production up to $\rho = -1.962$ and is highest under separable preferences. The highest reported share is for separable preferences $\alpha = 0.0$, and $\rho = -1.962$: around 0.38‰, a negligible fraction of households.

Table C.7: Fraction of households with negative labor earnings across preferences and hours complementarities

	α										
ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00
0.5	6.5e-05	1.3e-05	3.9e-06	1.2e-06	3.7e-07	1.1e-07	3.7e-08	6.7e-08	6.1e-08	6.0e-08	6.0e-08
0.02	2.1e-04	7.2e-05	3.1e-05	1.4e-05	7.6e-06	4.6e-06	3.3e-06	3.2e-06	1.8e-05	1.8e-05	1.8e-05
-0.46	3.2e-04	1.3e-04	6.6e-05	3.5e-05	2.0e-05	2.7e-05	2.3e-05	2.2e-05	2.1e-05	2.0e-05	1.8e-05
-1.962	3.8e-04	2.1e-04	1.2e-04	8.4e-05	6.0e-05	5.3e-05	1.5e-04	1.5e-04	1.4e-04	1.4e-04	1.4e-04
-7	1.9e-04	1.2e-04	7.4e-05	4.7e-05	4.6e-05	4.2e-05	1.5e-04	1.4e-04	1.4e-04	1.4e-04	1.4e-04

Note: The parameter α controls the degree of complementarity between consumption and hours, with $\alpha = 0$ corresponding to separable preferences and $\alpha = 1.0$ corresponding to GHH preferences. The parameter ρ controls complementarities between hours worked, with $\rho = 1.0$ corresponding to a standard linear aggregation, where hours are perfect substitutes, and values below zero indicating that hours are gross complements.

C.12. Distribution of Hours Worked in the Model and the Data

Data. I calculate the distribution of hours worked in the data using the usual hours worked variable (*uhrswork1*) from the CPS Outgoing Rotation Group. The sample includes working-age

women and men between 25 and 64 years old. Otherwise, the sample selection criteria are the same as in the baseline sample of Bick et al. (2022). I focus on individuals holding a single job, not enrolled in school, not self-employed, reporting positive earnings, and earning more than half a federal minimum wage. I discard observations with imputed or missing hours worked or earnings. I calculate group-specific mean usual weekly hours worked using CPS weights *wtfinl*.

Model. To translate the hours' distribution in the model into the data, I assume that the model mean hours correspond to the mean hours in the data. More concretely, in the model, I calculate the following object:

$$\bar{h} = \int_{x \in \mathcal{X}} \int_{a \in \mathcal{A}} h(a,x) D(a,x)$$

$\bar{h}$ typically takes values around 1, and I assume that $\bar{h}$ corresponds to the mean usual hours worked at the main job in CPS: 40.55 hours a week. Based on this assumption, for each point on the state space, I calculate hours worked $\bar{h}_{emp} = \frac{h_{model}}{\bar{h}_{model}} \cdot 40.55$, where $\bar{h}_{emp}$ denotes a data-consistent measure of hours in the model. Next, I assign individuals to bins as in Bick et al. (2022). I calculate the probability masses of agents within each bin and create histograms as in Figure 1.

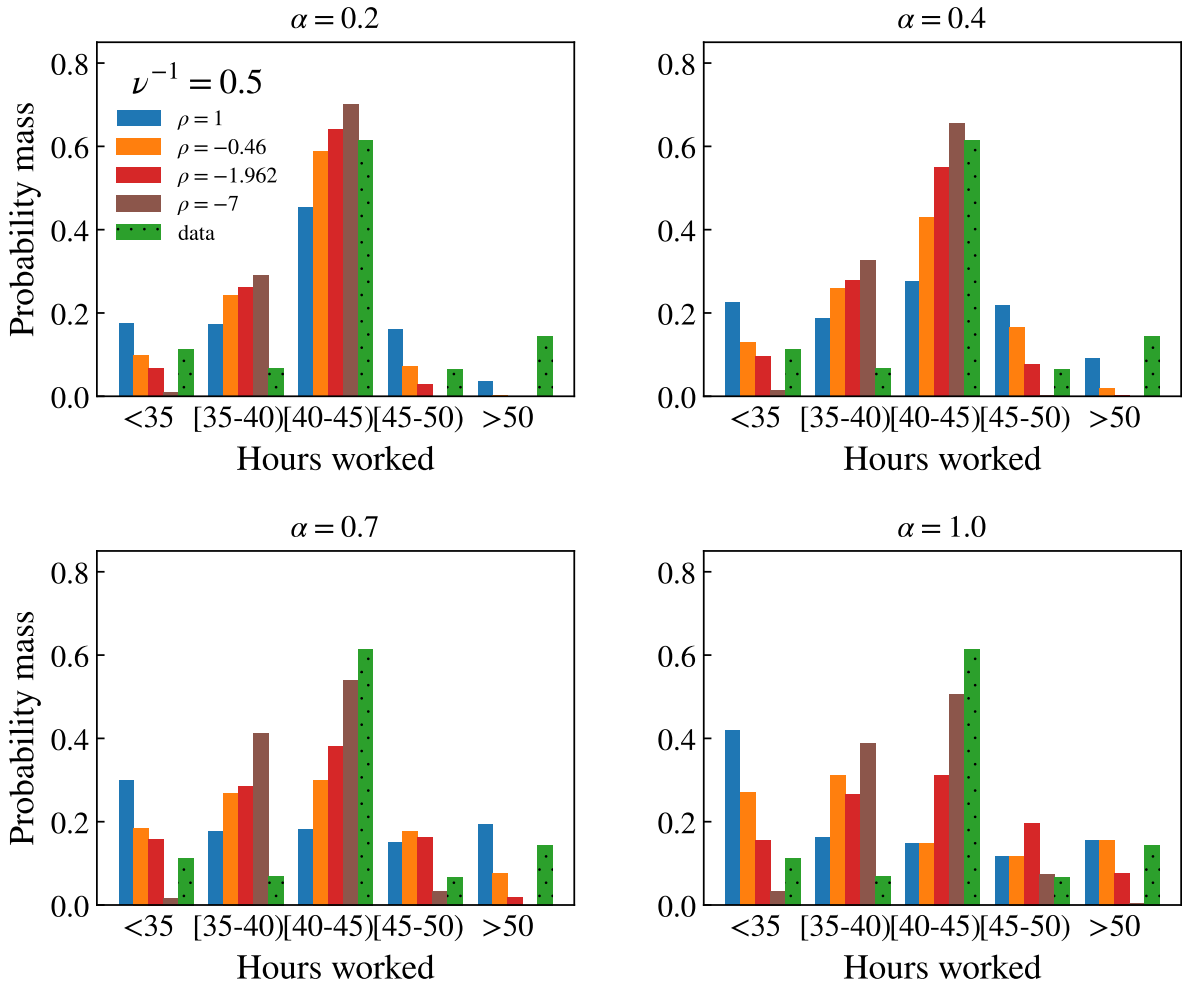
C.13. Steady-State Hours with GHH Plus Preferences

Figure C.1 shows that introducing complementarity between hours worked in production leads to a distribution of hours worked more concentrated around the mean. The effect is present regardless of the utility function assumed but tends to be stronger for low α calibrations, for example, for separable preferences.

C.14. Hours Complementarities and Wealth Distribution

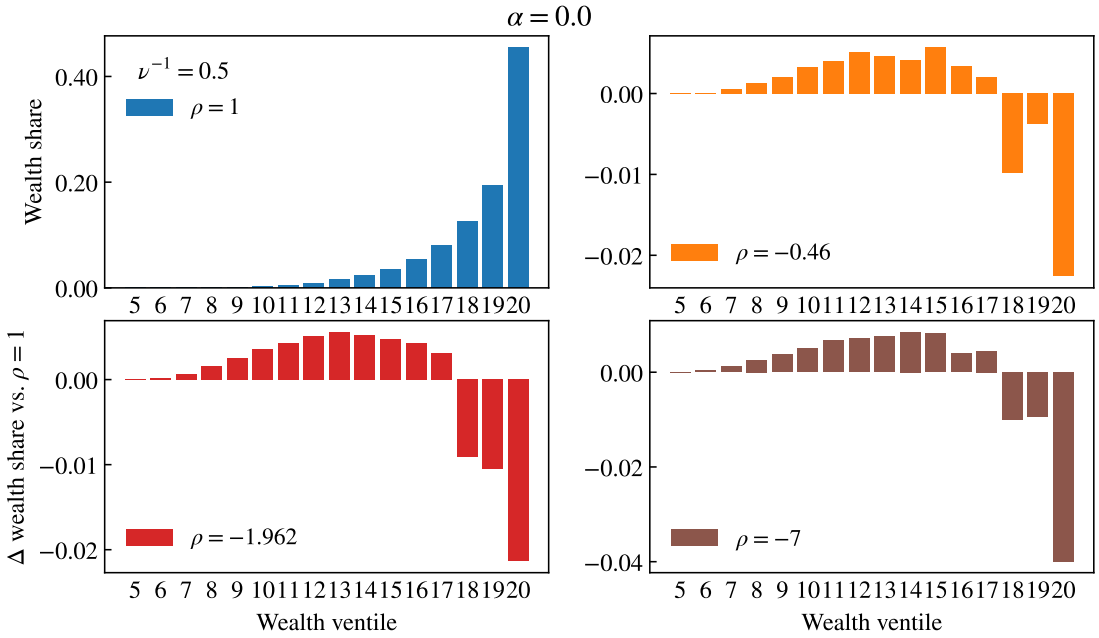
Stronger hours complementarities discourage households from working high hours. As a result, the wealth distribution becomes more equal with a smaller fraction of wealth held by the top 5% (see Figure C.2).

Figure C.1: Steady-state distribution of hours worked for different values of α



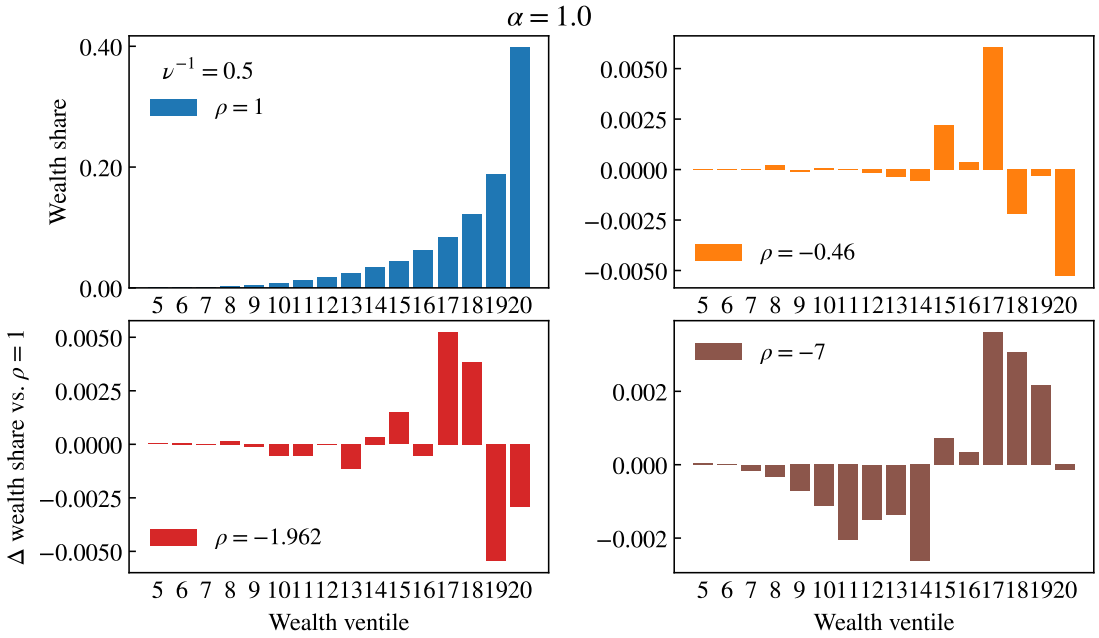
Note: The panels present the steady-state hours distributions in the models with different values of α and in the data. Different values of parameter ρ correspond to different degrees of hours complementarities in production. The lower the ρ, the more complementary hours worked are in production. Data source: see Figure 1.

Figure C.2: Hours complementarities and the steady-state wealth distribution with separable preferences



Note: The top-left panel presents the share of total wealth held by each wealth ventile in the baseline model with linear aggregation, $\rho = 1$. The other panels present the differences in wealth shares between the models with hours complementarities and the baseline model. Different values of parameter ρ correspond to different degrees of hours complementarities in production. The lower the ρ, the more complementary hours worked are in production.

Figure C.3: Hours complementarities and the steady-state wealth distribution with GHH preferences



Note: The top-left panel presents the share of total wealth held by each wealth ventile in the baseline model with linear aggregation, $\rho = 1$. The other panels present the differences in wealth shares between the models with hours complementarities and the baseline model. Different values of parameter ρ correspond to different degrees of hours complementarities in production. The lower the ρ, the more complementary hours worked are in production.

D. Ricardian Equivalence in the Sticky-Price Model

This appendix shows that the particular form of Ricardian equivalence that Auclert et al. (2023) establish for the baseline sticky-price model survives the introduction of hours complementarities in production. The result explains why the present-value fiscal multipliers presented in Table 7 are independent of the persistence of debt ρ_B in the sticky-price model.

Proposition A2. *Consider two alternative paths for debt and taxes—*$\{B_t, \tau_t^w\}_{t\geq 0}$ *and* $\{B_t^*, \tau_t^{w^*}\}_{t\geq 0}$*—that both satisfy the initial conditions* B_{-1}*,* $D_0(a,x)$*, and the government budget constraint* (31) *in every period given a government spending path* $\{G_t\}_{t\geq 0}$ *under constant transfers* T *and a constant real rate rule* $r_t^e = r_{ss}$*, as well as the no-Ponzi condition for debt*

$$\lim_{t\to\infty}(1+r_{ss})^{-t}B_t = \lim_{t\to\infty}(1+r_{ss})^{-t}B_t^* = 0.$$

Then, to first order, the two schemes support the same paths of consumption, hours worked, output, and household wealth, and hence the same fiscal multiplier.

Since transfers are constant, government spending is exogenous and follows an AR(1) process with persistence ρ_G, and debt follows a deficit financing rule with a parameter ρ_B, the linear labor tax rate τ_t^w varies over time ensuring that the government budget constraint holds period by period. The deficit-financing rule ensures that government debt eventually converges to its steady-state level for any $\rho_B < 1$, and the no-Ponzi condition holds.

Step 1: Household decisions depend on taxes and wages only through $(1-\tau_t^w)\tilde{w}_t$. Households take sequences of prices, taxes, and reference hours as given. Multiplying the wage-hours schedule (21) by hours worked and by one minus the labor tax rate gives after-tax labor earnings, which are homogeneous of degree one in $(1-\tau_t^w)\tilde{w}_t$

$$y(h, \tilde{w}_{it}^{\tau,x}, H_t) = (1-\tau_t^w)\tilde{w}_t x H_t \left(\frac{h^\rho}{\rho H_t^\rho} + \left(1-\frac{1}{\rho}\right)\right). \tag{D.78}$$

Likewise, marginal earnings that determine labor supply decisions are homogeneous of degree 1 in the after-tax aggregate wage index $(1-\tau_t^w)\tilde{w}_t$

$$y_h(h, \tilde{w}_{it}^{\tau,x}, H_t) = (1-\tau_t^w)\tilde{w}_t x \left(\left(\frac{h}{H_t}\right)^{\rho-1}\right). \tag{D.79}$$

Consequently, household decisions depend on taxes and the aggregate wage index only through the product of the two at any period t: $\{(1-\tau_t^w)\tilde{w}_t\}_{t\geq 0}$.

Step 2: Household policies depend on sequences of taxes, wages, and reference hours, for a given initial equity value. Under the constant real rate rule the ex-ante real rate equals r_{ss} in every period of the transition. The ex-post return therefore also equals r_{ss} in every period except $t = 0$, when the unanticipated shock revalues equity:

$$1 + r_t = \begin{cases} \frac{p_{ss}}{p_{ss}+B_{ss}} \frac{d_0+p_0}{p_{ss}} + \frac{B_{ss}}{p_{ss}+B_{ss}}(1 + r_{ss}) & \text{for } t = 0 \\ 1 + r_{ss} & \text{for } t \geq 1. \end{cases}$$

Combining this with (D.78), the aggregate hours supplied by the household block can be written as

$$H_t^s = \mathcal{H}_t \left(\left\{ (1 - \tau_s^w)\tilde{w}_s, H_s^d \right\}_{s \geq 0}, d_0 + p_0 \right).$$

Relative to the baseline model of Auclert et al. (2023), the sequence of reference hours $\{H_s^d\}_{s \geq 0}$ is the only additional argument.

Step 3: The revaluation effect depends only on aggregate labor and the aggregate wage bill. Iterating the pricing equation (30) forward, and imposing the no-Ponzi condition on firm equity $\lim_{t \to \infty} (1 + r_{ss})^{-t} p_t = 0$, yields

$$\begin{aligned} p_t &= \frac{d_{t+1} + p_{t+1}}{1 + r_t^e} \\ &= \sum_{s=0}^{\infty} \prod_{k=0}^{s} (1 + r_{t+k}^e)^{-1} d_{t+1+s} \\ \Rightarrow \quad p_0 &= \sum_{s=0}^{\infty} \prod_{k=0}^{s} (1 + r_k^e)^{-1} d_{s+1}. \end{aligned}$$

When the monetary policy follows the constant real rate rule, the ex-ante real rate is at the steady-state level in all periods of the transition. Therefore, the following holds

$$\begin{aligned} p_0 &= \sum_{t=0}^{\infty} (1 + r_{ss})^{-t-1} d_{t+1} \\ p_0 + d_0 &= \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} d_t. \end{aligned}$$

Appendix C.8 establishes that the aggregate wage bill equals $\tilde{w}_t N_t$. Using this together with the definition of dividends (23), using that the price adjustment term is zero to first order, and substituting for output from the aggregate production function (22) yields

$$p_0 + d_0 = \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (Y_t - \tilde{w}_t N_t)$$

$$= \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (zN_t - F - \tilde{w}_t N_t)$$
$$= \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} ((z - \tilde{w}_t)N_t - F) .$$

The result demonstrates that the sequences of aggregate labor $\{N_t\}_{t\geq 0}$ and of the aggregate wage bill $\{\tilde{w}_t N_t\}_{t\geq 0}$ pin down the revaluation effect. Iterating the government budget constraint (31) forward, and combining with the no-Ponzi condition for government debt from Proposition A2, yields

$$B_t = \sum_{s=0}^{\infty} \prod_{k=0}^{s} (1 + r^e_{t+k})^{-1} \left(\tau^w_{t+1+s} \tilde{w}_{t+1+s} N_{t+1+s} - G_{t+1+s} - T\right) .$$

Again, using that the monetary policy follows the constant real rate rule, and that the ex-ante real rate is constant at the steady-state level in all periods yields

$$B_t = \sum_{s=0}^{\infty} (1 + r_{ss})^{-(s+1)} \left(\tau^w_{t+1+s} \tilde{w}_{t+1+s} N_{t+1+s} - G_{t+1+s} - T\right) .$$

Substituting back in to the government budget constraint in period t yields

$$(1 + r_{ss})B_{t-1} = \sum_{s=0}^{\infty} (1 + r_{ss})^{-s} \left(\tau^w_{t+s} \tilde{w}_{t+s} N_{t+s} - G_{t+s} - T\right)$$
$$\Rightarrow \quad (1 + r_{ss})B_{-1} = \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} \left(\tau^w_t \tilde{w}_t N_t - G_t - T\right)$$

Step 4: Guess and verify that the two alternative paths generate the same allocations. The set of sequences $\{\tau^w_t, N_t, \tilde{w}_t\}_{t\geq 0}$ generates the baseline equilibrium path. Consider an alternative path and guess that the resulting equilibrium satisfies $N_t = N^*_t$ and $(1 - \tau^w_t)\tilde{w}_t = (1 - \tau^{w,*}_t)\tilde{w}^*_t$ for every t, so that the movements in the aggregate wage index exactly offset the movements in the tax rate. The guess implies that the after-tax wage bill is equal in both allocations in any period

$$(1 - \tau^w_t)\tilde{w}_t N_t = (1 - \tau^{w,*}_t)\tilde{w}^*_t N^*_t .$$

This immediately implies that the net present values of the after-tax wage bills are also equal

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (1 - \tau^w_t)\tilde{w}_t N_t = \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (1 - \tau^{w,*}_t)\tilde{w}^*_t N^*_t .$$

Because the two schemes share the initial debt B_{-1}, the spending path $\{G_t\}_{t\geq 0}$, and the transfer T, the present-value government budget constraint implies that the labor tax revenues are equal as well

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} \tau^w_t \tilde{w}_t N_t = \sum_{t=0}^{\infty} (1 + r_{ss})^{-t} \tau^{w,*}_t \tilde{w}^*_t N^*_t .$$

Combining the two implies that the present values of the pre-tax wage bills are equal in both allocations

$$\sum_{t=0}^{\infty}(1+r_{ss})^{-t}\tilde{w}_t N_t = \sum_{t=0}^{\infty}(1+r_{ss})^{-t}\tilde{w}_t^* N_t^*.$$

Since the conjecture was that $N_t = N_t^*$, Step 3 implies that the revaluation effects $p_0 + d_0$ are the same under both allocations.

Step 5: Aggregate labor supply and reference hours. Lastly, check that the inputs into the aggregate hours function $\mathcal{H}$ in Step 2 coincide across the two allocations. The guess implies that the after-tax wages $(1-\tau_t^w)\tilde{w}_t$ are equal. The revaluation effects $d_0 + p_0$ are equal by combining the definition in Step 3 with the equality of wage bills derived in Step 4. Appendix C.7 shows that when average idiosyncratic productivity is normalized to one and all households work, reference hours coincide with aggregate labor, $H_t = N_t$, so the guess $N_t = N_t^*$ delivers $H_t = H_t^*$ directly. Households therefore face the same sequences of inputs and start from the same distribution $D_0(a,x)$ along both paths. Therefore, their policy functions, and hence the entire path of the distribution, are identical. Aggregating those policies returns $N_t^s = N_t^{s,*}$, which verifies the guess $N_t = N_t^*$.

Step 6: Market clearing. The equality of the aggregate labor sequences and the fact that the labor supply policies coincide implies that the labor market clears in both allocations. Since the aggregate production function is linear in labor (22) and consumption policies coincide, therefore the goods market clears under both allocations as well. Since the same sequences of inputs are provided to the household block, the consumption sequences are also equal, $C_t = C_t^*$, so goods market clearing holds in both allocations up to the price adjustment cost. The paths considered are first-order, therefore the Rotemberg adjustment cost does not affect the goods market clearing. Absent the Rotemberg cost, all equalities hold exactly without invoking the first-order argument. The differences in the paths of the aggregate wage index $\tilde{w}_t$ imply differences in marginal costs and inflation. Since the central bank ensures a constant real rate, the inflation differences are fully offset by the nominal rates set by the central bank.

The asset market clearing holds as well because the savings policies coincide and the differences in debt holdings are fully offset by differences in the equity value. Adding the forward-iterated equity price and the forward-iterated government budget constraint at an arbitrary date t, and ignoring the second-order adjustment cost, gives

$$p_t + B_t = \sum_{s=1}^{\infty}(1+r_{ss})^{-s}\left(Y_{t+s} - (1-\tau_{t+s}^w)\tilde{w}_{t+s}N_{t+s} - G_{t+s} - T\right).$$

All sequences on the right hand side coincide across both allocations, hence total liquid asset supply coincides as well. Since the total asset demand is the same as well because the household inputs are the same, the asset market clears in both allocations.

The firm's cash flows are discounted using the same real rate that government debt pays. Debt reductions that follow tax increases are exactly offset by the increases in equity value associated with lower future taxes and, therefore, lower future pre-tax wages. While the pre-tax real wage, the markup, inflation, and the equity price move, the objects that matter for household optimization—the real rate and the post-tax wage—do not. Therefore, consumption, hours worked, and output are invariant to paths of government debt. The key assumption is that the wage can adjust flexibly to any variation in debt and taxes. That is why the sticky-wage model of Auclert et al. (2023) breaks the equivalence, as the last row of Table 7 shows.

E. Sticky-Wage Model with Heterogeneous Hours and Hours Complementarities

This section presents the sticky-wage HANK model with heterogeneous hours and complementarities between workers' hours. The framework builds on the heterogeneous-hours HANK model in Gerke et al. (2024). I extend their framework to complementarities between workers' hours in the labor aggregation, as well as features necessary to match a quarterly MPC of 0.25, as well as the US statistics on asset supply from the US.

Households

Households supply hours to a continuum of unions indexed by k

$$h_{it} = \int h_{ikt}\, dk.$$

Households solve the same problem as described in (24) with the difference that hours h_{ikt} are chosen jointly with the unions.

However, contrary to the uniform-hours setup, unions take into account every household's individual circumstances when setting the hours. Therefore, the hours determination process can be viewed as a joint decision between the union and the household, where the optimality condition for a household's hours reflects the utility margins of the household, as well as the market power and labor aggregation effects that matter in collective bargaining.

Firms

Final-goods producers are the same as in the sticky-price model. The intermediate-goods producers indexed by j use a production technology that is linear in the labor input

$$y_{jt} = z n_{jt} - F.$$

Prices are flexible, and all firms produce the same output $Y_t = y_{jt}$ and choose the same labor input $n_{jt} = N_t$.

A Competitive Labor Packer

There is a competitive labor packer that purchases union-specific tasks and combines them into aggregate labor services that can be sold to the intermediate-goods producers. Union-specific

tasks are imperfect substitutes which gives market power and ability to set prices to the unions. The price of an aggregate labor service is $\tilde{w}_t$, and the price of a union-specific task is $\tilde{w}_{kt}$. Labor is aggregated using CES technology

$$N_t = \left(\int_k N_{kt}^{\frac{\epsilon_w - 1}{\epsilon_w}} \, dk \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{E.80}$$

where ϵ_w denotes the elasticity of substitution between union-specific tasks. The packer chooses how much of each task to purchase to maximize profits, and its first-order condition yields a standard CES demand schedule

$$N_{kt} = \left(\frac{\tilde{w}_{kt}}{\tilde{w}_t} \right)^{-\epsilon_w} N_t.$$

Union

The union consists of two decision-making bodies. The first body is the production unit which aggregates individual labor supply into tasks, which are then sold to a competitive labor packer. The second body is the collective-bargaining unit which decides how much labor to hire and what task-specific wage $\tilde{w}_{kt}$ to set to maximize the average welfare of its members. The labor supply decisions account for individual households' circumstances, allowing their labor supply to adjust in response to idiosyncratic changes in economic circumstances, including unanticipated wealth shocks. Therefore, the MPE can be positive, contrary to the framework of Auclert et al. (2023), where the MPE is set to zero by construction.

The Labor Market and the Union's Problem

There is a continuum of labor unions indexed by k. Each union produces a task N_{kt}. Tasks are imperfect substitutes, which generates market power on the part of unions:

$$N_t = \left(\int_k N_{kt}^{\frac{\epsilon_w - 1}{\epsilon_w}} \, dk \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}.$$

A competitive labor packer purchases labor services from unions, and the solution of its problem is the following standard CES demand schedule:

$$N_{kt} = \left(\frac{\tilde{w}_{kt}}{\tilde{w}_t} \right)^{-\epsilon_w} N_t.$$

Each union has a representative fraction of the workforce as its members. It chooses hours worked h_{ikt} for all its members, and sells them to an internal labor aggregation unit. The labor aggregation

technology allows for potential complementarities between workers' hours

$$N_{kt} = \left(\int x_{it} h_{i,k,t}^{\rho} di \right)^{\frac{1}{\rho}} \left(\int x_{it}\, di \right)^{1-\frac{1}{\rho}},$$

where $\rho = \frac{\sigma_h - 1}{\sigma_h}$, and σ_h equals the elasticity of substitution between workers' hours. The internal production unit meets a workforce that the union sends to it and faces a cost of discarding workers which is sufficiently high to ensure that no workers are discarded.

The Union's Labor Aggregation Unit

I will start by deriving the internal wage schedule from the problem of the internal labor aggregation unit. The labor aggregation unit posts internal demand $\{\mu(h,x)\}_{h\in\mathcal{H},x\in\mathcal{X}}$ for every worker type characterized by hours h and productivity x. The demand is chosen to maximize the unit's profit, where the revenues come from selling the aggregated task product to the union and the cost is paying each worker a salary $w(h,x)h$. The internal price of the aggregated task equals its market price $\tilde{w}_{kt}$.

$$\max_{\{\mu(h,x)\}_{h\in\mathcal{H},x\in\mathcal{X}}} \tilde{w}_{kt} N_{kt} - \int_{h\in\mathcal{H}} \int_{x\in\mathcal{X}} w(h,x) h \mu(h,x)\, dh\, dx$$
$$\text{s.t. } N_{kt} = \left(\int_{h\in\mathcal{H}} \int_{x\in\mathcal{X}} x h^{\rho} \mu(h,x)\, dh\, dx \right)^{\frac{1}{\rho}} \left(\int_{h\in\mathcal{H}} \int_{x\in\mathcal{X}} x \mu(h,x)\, dh\, dx \right)^{1-\frac{1}{\rho}}$$

The first-order condition of the production unit is the following wage-hours menu

$$w_{kt}(h,x) = \tilde{w}_{kt} x H_{kt} \left(\frac{h^{\rho-1}}{\rho H_{kt}^{\rho}} + \frac{1}{h}\left(1 - \frac{1}{\rho}\right) \right), \tag{E.81}$$

where H_{kt} denotes union-specific reference hours, the point at which the wage-hours schedule peaks.

Union-level aggregate labor is homogeneous of degree one in the set of worker measures $\{\mu(h,x)\}$. Therefore, it makes zero profits at the interior solution.

The unit hires all of the union's members, whose average idiosyncratic productivity equals unity,

$$\int_{h\in\mathcal{H}} \int_{x\in\mathcal{X}} x \mu(h,x)\, dh\, dx = \int x_{it}\, di = 1,$$

so reference hours also equal union-specific aggregate hours

$$H_{kt} = N_{kt}.$$

The Collective-Bargaining Unit

The collective-bargining unit is the decision-making body of the union that is not engaged in any productive activities. Its role is to choose the price of the task k produced: $\tilde{w}_{kt}$, and hours worked of all member households $\{h_{ikt}\}_{i\in[0,1]}$ to maximize its members' average welfare subject to the quadratic utility cost of adjusting wages, and demand and supply constraints on labor.

The union takes the wage schedule offered by the production unit as given and only determines its union-level components: the task-specific wage $\tilde{w}_{kt}$ and reference hours H_{kt}. The adjustments of the union-level wage component are subject to a quadratic utility cost. Thus, unions choose the hours of each member $\{h_{ik,t+s}\}_{i\in[0,1],s\geq 0}$, the total quantity of task k produced, $\{N_{k,t+s}\}_{s\geq 0}$, union-specific referece hours, $\{H_{k,t+s}\}_{s\geq 0}$, and wages $\{\tilde{w}_{k,t+s}\}_{s\geq 0}$ to maximize the average welfare of its members:

$$\sum_{s\geq 0}\beta^{t+s}\left[\int U\left(c_{i,t+s},\int_k h_{i,k,t+s}dk\right)di-\frac{\psi}{2}\left(\log\left(\frac{\tilde{w}_{k,t+s}}{\tilde{w}_{k,t+s-1}}(1+\pi_{t+s})\right)\right)^2\right].$$

They respect their members' budget constraints

$$c_{i,t+s}+a_{i,t+s+1}=(1+r_{t+s})a_{i,t+s}+(1-\tau^w_{t+s})\int \tilde{w}_{k,t+s}x_{i,t+s}H_{k,t+s}\left(\frac{h^\rho_{ik,t+s}}{\rho H^\rho_{k,t+s}}+1-\frac{1}{\rho}\right)dk+T.$$

Unions realize that choosing individual hours affects reference hours which enter its members' budget constraints[52]

$$H_{kt}=\left(\int x_{it}h^\rho_{ikt}di\right)^{\frac{1}{\rho}}\left(\int x_{it}\,di\right)^{-\frac{1}{\rho}}.$$

When choosing how much labor to hire, unions need to satisfy the CES demand for aggregate labor

$$N_{kt}=\left(\frac{\tilde{w}_{kt}}{\tilde{w}_t}\right)^{-\epsilon_w}N_t.$$

Hours that unions choose cannot be negative for any household i

$$h_{ikt}\geq 0.$$

I assume a symmetric equilibrium with $\tilde{w}_{kt}=\tilde{w}_t$ and hence $N_{kt}=N_t$. The union's optimality conditions imply the following New Keynesian Wage Phillips Curve

$$\log\left(1+\pi^w_t\right)=\kappa_w N_t\left[-\int\left(\frac{h_{it}}{H_t}\right)^{1-\rho}\left(U_h(c_{it},h_{it})+\xi_{i,t}\right)di\right.$$

[52]The derivative is taken with respect to the union's hours schedule and therefore perturbs a positive mass of members. With M members of mass $1/M$, one member's hours change H_{kt} by $x_{it}(h_{ikt}/H_{kt})^{\rho-1}/M$, and all M members are affected, so the product does not vanish as $M\to\infty$. An idiosyncratic shock to one member also changes H_{kt} at rate $1/M$, but is not multiplied by the number of members affected, so reference hours are unchanged in the limit. This is discussed at the end of the section.

$$- (1-\tau_t^w)\tilde{w}_t \int \left[\left(\frac{\epsilon_w - 1}{\epsilon_w} - \rho\right)\Phi_{it} + \rho\right] x_{it} U_c(c_{it}, h_{it})\, di \Bigg]$$
$$+ \beta \log\left(1 + \pi_{t+1}^w\right), \tag{E.82}$$

where $\xi_{i,t}$ denotes the Lagrange multiplier on household i's labor supply non-negativity constraint, and $\Phi_{i,t}$ denotes the household-hours-dependent component of labor earnings

$$\Phi_{i,t} \equiv \frac{1}{\rho}\left(\frac{h_{i,t}}{H_t}\right)^{\rho} + 1 - \frac{1}{\rho}.$$

The optimal labor supply of each household satisfies

$$\begin{aligned} -U_h(c_{it}, h_{it}) &= \lambda_{it}(1-\tau_t^w)\tilde{w}_t x_{it}\left(\frac{h_{it}}{H_t}\right)^{\rho-1} - \lambda_t^w x_{it}\left(\frac{h_{it}}{H_t}\right)^{\rho-1} \\ &= \left(\lambda_{it}(1-\tau_t^w)\tilde{w}_t - \lambda_t^w\right) x_{it}\left(\frac{h_{it}}{H_t}\right)^{\rho-1}, \end{aligned}$$

where λ_t^w is a sum of Lagrange multipliers on the CES demand and reference hours constraints. Basically the union internalizes the household's objective function but corrects optimal labor supply to account for the effect it has on average welfare through the wage required to generate a given CES demand, as well as the effect it has on reference hours in the union, which affect all workers' labor earnings.

E.1. Union's Optimality Conditions

The Lagrangian corresponding to these objective and constraint functions is the following:

$$\begin{aligned} &\mathcal{L}\left(\{\tilde{w}_{k,t+s}, N_{k,t+s}, H_{k,t+s}, (h_{i,k,t+s})_{i\in[0,1]}, c_{i,t+s}, a_{i,t+s}\}_{s\geq 0}; \{\lambda\}\right) \\ &= \sum_{s\geq 0}\beta^{t+s}\left[\int U\left(c_{i,t+s}, \int_k h_{i,k,t+s}dk\right) di - \frac{\psi}{2}\left(\log\left(\frac{\tilde{w}_{k,t+s}}{\tilde{w}_{k,t+s-1}}(1+\pi_{t+s})\right)\right)^2\right] \\ &+ \sum_{s\geq 0}\beta^{t+s}\lambda_{k,t+s}^N\left(\left(\frac{\tilde{w}_{k,t+s}}{\tilde{w}_{t+s}}\right)^{-\epsilon_w} N_{t+s} - N_{k,t+s}\right) \\ &+ \sum_{s\geq 0}\beta^{t+s}\Bigg[\int \lambda_{2,i,t+s}\Big((1+r_{t+s})a_{i,t+s} + (1-\tau_{t+s}^w)\int \tilde{w}_{k,t+s}x_{i,t+s}H_{k,t+s}\left(\frac{h_{ik,t+s}^{\rho}}{\rho H_{k,t+s}^{\rho}} + 1 - \frac{1}{\rho}\right)dk \\ &\qquad + T_{t+s} - c_{i,t+s} - a_{i,t+s+1}\Big)\, di\Bigg] \\ &+ \sum_{s\geq 0}\beta^{t+s}\lambda_{3,k,t+s}\left(N_{k,t+s} - \left(\int x_{i,t+s}h_{i,k,t+s}^{\rho}\, di\right)^{\frac{1}{\rho}}\left(\int x_{i,t+s}\, di\right)^{1-\frac{1}{\rho}}\right) \\ &+ \sum_{s\geq 0}\beta^{t+s}\lambda_{k,t+s}^H\left(H_{k,t+s} - \left(\int x_{i,t+s}h_{i,k,t+s}^{\rho}\, di\right)^{\frac{1}{\rho}}\left(\int x_{i,t+s}\, di\right)^{-\frac{1}{\rho}}\right) \end{aligned}$$

$$+\sum_{s\geq 0}\beta^{t+s}\left[\int \xi_{i,t+s}h_{i,t+s}\,di\right].$$

The first-order conditions of the union's problem are the following:

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial\tilde{w}_{k,t}}: \quad & -\psi\beta^t\log\left(\frac{\tilde{w}_{k,t}}{\tilde{w}_{k,t-1}}(1+\pi_t)\right)\frac{1}{\tilde{w}_{k,t}} \\
& +\psi\beta^{t+1}\log\left(\frac{\tilde{w}_{k,t+1}}{\tilde{w}_{k,t}}(1+\pi_{t+1})\right)\frac{1}{\tilde{w}_{k,t}} \\
& -\epsilon_w\beta^t\lambda_t^N\tilde{w}_{k,t}^{-\epsilon_w-1}\tilde{w}_t^{\epsilon_w}N_t \\
& +\beta^t(1-\tau_t^w)\int\lambda_{2,i,t}x_{it}H_{kt}\left(\frac{h_{ikt}^{\rho}}{\rho H_{kt}^{\rho}}+1-\frac{1}{\rho}\right)di \\
& =0 \qquad\qquad (E.83)\\
\frac{\partial\mathcal{L}}{\partial N_{kt}}: \quad & -\beta^t\lambda_t^N+\beta^t\lambda_{3,t}=0 \\
\frac{\partial\mathcal{L}}{\partial H_{kt}}: \quad & \beta^t\int\lambda_{2,i,t}(1-\tau_t^w)\tilde{w}_{k,t}x_{it}\left(1-\frac{1}{\rho}\right)\left(1-\left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho}\right)di \\
& +\beta^t\lambda_t^H \\
& =0 \\
\frac{\partial\mathcal{L}}{\partial h_{ikt}}: \quad & \beta^t U_h(c_{it},h_{it}) \\
& +\beta^t\lambda_{2,i,t}(1-\tau_t^w)\tilde{w}_{k,t}x_{it}H_{kt}^{1-\rho}h_{ikt}^{\rho-1} \\
& -\beta^t\lambda_{3,t}x_{it}h_{ikt}^{\rho-1}H_{kt}^{1-\rho} \\
& -\beta^t\lambda_t^H x_{it}h_{ikt}^{\rho-1}H_{kt}^{1-\rho}X^{-1} \\
& +\beta^t\xi_{i,t} \\
& =0,
\end{aligned}$$

where $X=\int x_{it}di=1$. The multipliers on the CES demand and resource constraints on aggregate hours are equal

$$\lambda_{3,t}=\lambda_t^N.$$

The first-order condition with respect to reference hours, together with $\lambda_{2,i,t}=U_c(c_{it},h_{it})$ (the household's and union's multiplier on the budget constraint overlap), yields the following Lagrange multiplier on the reference-hours constraint

$$\lambda_t^H=-\int U_c(c_{it},h_{it})(1-\tau_t^w)\tilde{w}_{k,t}x_{it}\left(1-\frac{1}{\rho}\right)\left(1-\left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho}\right)di \qquad (E.84)$$

The first-order condition with respect to individual household's hours reads

$$-U_h(c_{it},h_{it})=U_c(c_{it},h_{it})(1-\tau_t^w)\tilde{w}_{kt}x_{it}\left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho-1}$$

$$
\begin{aligned}
&- \lambda_{3,t} x_{it} \left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho-1} \\
&- \lambda_t^H x_{it} \left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho-1} \\
&+ \xi_{i,t}
\end{aligned} \tag{E.85}
$$

Substituting for $\lambda_{3,t}$ from the aggregate-hours condition and for λ_t^H from the condition for reference hours yields

$$
\begin{aligned}
-U_h(c_{it}, h_{it}) = {} & U_c(c_{it}, h_{it})(1-\tau_t^w)\tilde{w}_{kt} x_{it} \left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho-1} \\
& - \lambda_t^N x_{it} \left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho-1} \\
& + x_{it} \left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho-1} \int U_c(c_{it}, h_{it})(1-\tau_t^w)\tilde{w}_{k,t} x_{it} \left(1-\frac{1}{\rho}\right)\left(1-\left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho}\right) di \\
& + \xi_{i,t},
\end{aligned}
$$

which can equivalently be written as

$$
\begin{aligned}
-U_h(c_{it}, h_{it}) = {} & \Bigg[(1-\tau_t^w)\tilde{w}_{kt} U_c(c_{it}, h_{it}) - \lambda_t^N \\
& + \int U_c(c_{it}, h_{it})(1-\tau_t^w)\tilde{w}_{k,t} x_{it} \left(1-\frac{1}{\rho}\right)\left(1-\left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho}\right) di\Bigg] x_{it} \left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho-1} \\
& + \xi_{i,t}.
\end{aligned} \tag{E.86}
$$

The optimality condition on hours and consumption is similar to the sticky-price flexible-wage model but includes two additional terms. First, the Lagrange multiplier on the demand constraint λ_t^N reduces the optimal labor supply. Second, since unions choose hours for all their members they internalize the effect that individual hours have on reference hours, which changes the incentives to work depending on individual hours.

E.2. Derivation of the New Keynesian Wage Phillips Curve

The New Keynesian Wage Phillips Curve can be derived from the union's optimality condition with respect to the union-level wage component $\tilde{w}_{kt}$ (E.83), which can be expressed as:

$$\begin{aligned}\log\left(1+\pi_t^w\right) = &\frac{1}{\psi}(1-\tau_t^w)\tilde{w}_{kt}\int U_c(c_{it},h_{it})x_{it}H_{kt}\left(\frac{h_{ikt}^{\rho}}{\rho H_{kt}^{\rho}}+1-\frac{1}{\rho}\right)di\\ &-\frac{\epsilon_w}{\psi}\lambda_t^N\left(\frac{\tilde{w}_{kt}}{\tilde{w}_t}\right)^{-\epsilon_w}N_t\\ &+\beta\log\left(1+\pi_{t+1}^w\right).\end{aligned}$$

Equation (E.86) is a continuum of conditions sharing a single multiplier. Rearranging it gives

$$\begin{aligned}\lambda_t^N = &(1-\tau_t^w)\tilde{w}_{kt}U_c(c_{it},h_{it})\\ &+\int U_c(c_{it},h_{it})(1-\tau_t^w)\tilde{w}_{k,t}x_{it}\left(1-\frac{1}{\rho}\right)\left(1-\left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho}\right)di\\ &+\frac{U_h(c_{it},h_{it})}{x_{it}h_{ikt}^{\rho-1}H_{kt}^{1-\rho}}\\ &+\frac{\xi_{i,t}}{x_{it}h_{ikt}^{\rho-1}H_{kt}^{1-\rho}}.\end{aligned}$$

Multiply through by idiosyncratic productivity x_{it}

$$\begin{aligned}x_{it}\lambda_t^N = &(1-\tau_t^w)\tilde{w}_{kt}x_{it}U_c(c_{it},h_{it})\\ &+x_{it}\int U_c(c_{it},h_{it})(1-\tau_t^w)\tilde{w}_{k,t}x_{it}\left(1-\frac{1}{\rho}\right)\left(1-\left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho}\right)di\\ &+\left(\frac{h_{ikt}}{H_{kt}}\right)^{1-\rho}U_h(c_{it},h_{it})\\ &+\left(\frac{h_{ikt}}{H_{kt}}\right)^{1-\rho}\xi_{i,t},\end{aligned}$$

and integrate over individuals

$$\begin{aligned}\lambda_t^N = &(1-\tau_t^w)\tilde{w}_{kt}\int x_{it}U_c(c_{it},h_{it})\,di\\ &+\int U_c(c_{it},h_{it})(1-\tau_t^w)\tilde{w}_{k,t}x_{it}\left(1-\frac{1}{\rho}\right)\left(1-\left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho}\right)di\\ &+\int\left(\frac{h_{ikt}}{H_{kt}}\right)^{1-\rho}U_h(c_{it},h_{it})\,di\\ &+\int\left(\frac{h_{ikt}}{H_{kt}}\right)^{1-\rho}\xi_{i,t}\,di.\end{aligned}\tag{E.87}$$

Assume a symmetric equilibrium, where all unions choose the same wage $\tilde{w}_t = \tilde{w}_{kt}$, and therefore, union-specific labor aggregates are equal $N_{kt} = N_t$. Substituting the Lagrange multiplier λ_t^N into the NKWPC yields:

$$\begin{aligned}
\log\left(1+\pi_t^w\right) = &\frac{1}{\psi}(1-\tau_t^w)\tilde{w}_t \int U_c(c_{it},h_{it})x_{it}H_{kt}\left(\frac{h_{ikt}^{\rho}}{\rho H_{kt}^{\rho}}+1-\frac{1}{\rho}\right) di \\
&-\frac{\epsilon_w}{\psi}N_t(1-\tau_t^w)\tilde{w}_t\int x_{it}U_c(c_{it},h_{it})\,di \\
&-\frac{\epsilon_w}{\psi}N_t\int U_c(c_{it},h_{it})(1-\tau_t^w)\tilde{w}_t x_{it}\left(1-\frac{1}{\rho}\right)\left(1-\left(\frac{h_{ikt}}{H_{kt}}\right)^{\rho}\right) di \\
&-\frac{\epsilon_w}{\psi}N_t\int\left(\frac{h_{ikt}}{H_{kt}}\right)^{1-\rho}U_h(c_{it},h_{it})\,di \\
&-\frac{\epsilon_w}{\psi}N_t\int\left(\frac{h_{ikt}}{H_{kt}}\right)^{1-\rho}\xi_{i,t}\,di \\
&+\beta\log\left(1+\pi_{t+1}^w\right).
\end{aligned}$$

To make further progress, note that $H_{kt} = H_t = N_t$ because average idiosyncratic productivity is unity. If reference hours are equal in all unions, then optimal hours are also the same $h_{ikt} = h_{it}$. Denote the slope of the Phillips curve $\kappa_w = \epsilon_w/\psi$ to get

$$\begin{aligned}
\log\left(1+\pi_t^w\right) = \kappa_w N_t\Bigg[&-\int\left(\frac{h_{it}}{H_t}\right)^{1-\rho}\left(U_h(c_{it},h_{it})+\xi_{i,t}\right)\,di \\
&-\left[1+\left(1-\frac{1}{\rho}\right)\left(\frac{\epsilon_w-1}{\epsilon_w}\right)\right](1-\tau_t^w)\tilde{w}_t\int U_c(c_{it},h_{it})x_{it}\,di \\
&-\frac{1}{\rho}\left(\frac{\epsilon_w-1}{\epsilon_w}\right)(1-\tau_t^w)\tilde{w}_t\int U_c(c_{it},h_{it})x_{it}\left(\frac{h_{it}}{H_t}\right)^{\rho}di \\
&+(1-\tau_t^w)\tilde{w}_t\int U_c(c_{it},h_{it})x_{it}\left(\frac{h_{it}}{H_t}\right)^{\rho}di\Bigg] \\
&+\beta\log\left(1+\pi_{t+1}^w\right)
\end{aligned}$$

The expression can be further simplified by accumulating terms inside the integral

$$\begin{aligned}
\log\left(1+\pi_t^w\right) = \kappa_w N_t\Bigg[&-\int\left(\frac{h_{it}}{H_t}\right)^{1-\rho}\left(U_h(c_{it},h_{it})+\xi_{i,t}\right)\,di \\
&-(1-\tau_t^w)\tilde{w}_t\int\left[\left(\frac{\epsilon_w-1}{\epsilon_w}-\rho\right)\Phi_{it}+\rho\right]x_{it}U_c(c_{it},h_{it})\,di\Bigg] \\
&+\beta\log\left(1+\pi_{t+1}^w\right)
\end{aligned} \tag{E.88}$$

where

$$\Phi_{it} = \frac{1}{\rho}\left(\frac{h_{it}}{H_t}\right)^{\rho}+1-\frac{1}{\rho}.$$

E.3. Lagrange Multiplier on the Reference Hours Constraints

In this section, I show that the following equation holds:

$$\lambda_t^H = -\tilde{w}_t^\tau (1-\rho) \operatorname{Cov}_x \left(U_c, \Phi\right).$$

Equation (E.84) provides an expression for the Lagrange multiplier on the reference hours constraint.

$$\lambda_t^H = -\int U_c(c_{it}, h_{it}) \tilde{w}_{it}^{\tau,x} \left(1 - \frac{1}{\rho}\right) \left(1 - \left(\frac{h_{it}}{H_t}\right)^\rho\right) di$$

With the following notation

$$\Phi_{it} = \frac{1}{\rho}\left(\frac{h_{it}}{H_t}\right)^\rho + 1 - \frac{1}{\rho},$$

one can equivalently express the multiplier in the following manner

$$\begin{aligned}
\lambda_t^H &= -\tilde{w}_t^\tau \operatorname{Cov}_x \left(U_c, \Phi\right) - \tilde{w}_t^\tau \mathop{\mathbb{E}}_x \left[U_c\right] \mathop{\mathbb{E}}_x \left[\Phi\right] + \tilde{w}_t^\tau \int U_c(c_{it}, h_{it}) x_{it} \left(\frac{h_{it}}{H_t}\right)^\rho di \\
&= -\tilde{w}_t^\tau \operatorname{Cov}_x \left(U_c, \Phi\right) - \tilde{w}_t^\tau \mathop{\mathbb{E}}_x \left[U_c\right] + \tilde{w}_t^\tau \operatorname{Cov}_x \left(U_c, \left(\frac{h}{H}\right)^\rho\right) + \tilde{w}_t^\tau \mathop{\mathbb{E}}_x \left[U_c\right] \\
&= -\tilde{w}_t^\tau \operatorname{Cov}_x \left(U_c, \Phi\right) + \tilde{w}_t^\tau \operatorname{Cov}_x \left(U_c, \left(\frac{h}{H}\right)^\rho\right) \\
&= -\tilde{w}_t^\tau \operatorname{Cov}_x \left(U_c, \Phi\right) + \tilde{w}_t^\tau \operatorname{Cov}_x \left(U_c, \rho\Phi\right) \\
&= -\underbrace{\tilde{w}_t^\tau (1-\rho)}_{\geq 0} \operatorname{Cov}_x \left(U_c, \Phi\right)
\end{aligned}$$

E.4. Perfect Complementarities and the New Keynesian Wage Phillips Curve

Even though individual households' hours converge to reference hours, the model's New Keynesian Wage Phillips Curve is different from the corresponding equation in the canonical sticky-wage model. The difference boils down to the concern that unions in my model have for individual workers' hours, which they do not in the canonical model. In the canonical model, a union only chooses aggregate hours which all households have to work. In the heterogeneous-hours model with hours complementarities, the union chooses both individual households' hours and the reference hours. Therefore, the union in my model accounts for the effect reference hours have on individual earnings, and the union in the canonical model does not.

Formally, the difference comes down to the component of the NKWPC that reflects the effect reference hours have on earnings,

$$y_H = \tilde{w}_{it}^{\tau,x} \left(1 - \frac{1}{\rho}\right) \left[1 - \left(\frac{h_{it}}{H_t}\right)^\rho\right],$$

which enters the New Keynesian Wage Phillips Curve through the same bracket,

$$\lim_{\rho\to-\infty} \rho\,(1-\Phi_{it}) = 1 - \lim_{\rho\to-\infty}\left(\frac{h_{it}}{H_t}\right)^{\rho}.$$

The key observation is that even though individual hours converge to reference hours, the same ratio raised to the power ρ does not:

$$\begin{aligned}\lim_{\rho\to-\infty}\left(\frac{h_{it}}{H_t}\right)^{\rho} &= \lim_{\rho\to-\infty}\frac{h_{it}}{H_t}\frac{\varphi h_{it}^{\nu}\,(1-\alpha+\alpha U_c(c_{it},h_{it}))}{x_{it}\left(U_c(c_{it},h_{it})(1-\tau_t^w)\tilde{w}_t-\lambda_t^w\right)}\\ &= \lim_{\rho\to-\infty}\frac{h_{it}}{H_t}\cdot\lim_{\rho\to-\infty}\frac{\varphi h_{it}^{\nu}\,(1-\alpha+\alpha U_c(c_{it},h_{it}))}{x_{it}\left(U_c(c_{it},h_{it})(1-\tau_t^w)\tilde{w}_t-\lambda_t^w\right)}\\ &= \frac{\varphi H_t^{\nu}\,(1-\alpha+\alpha U_c(c_{it},H_t))}{x_{it}\left(U_c(c_{it},H_t)(1-\tau_t^w)\tilde{w}_t-\lambda_t^w\right)}.\end{aligned}$$

If the sum of the union's multipliers $\lambda^w < 0$ is negative, then the limit exists and is finite. I will denote the limit as

$$\chi_{it}^{\infty} \equiv \lim_{\rho\to-\infty}\left(\frac{h_{it}}{H_t}\right)^{\rho}.$$

The definition of reference hours implies $\int x_{it}\,(h_{it}/H_t)^{\rho}\;di = 1$ at every ρ, so the limit averages to one under the productivity-weighted measure,

$$\int x_{it}\chi_{it}^{\infty}\,di = 1.$$

What matters for the New Keynesian Wage Phillips Curve is therefore not the level of the limit but its dispersion across households. As a result, the New Keynesian Wage Phillips Curve does not reduce to the NKWPC in the canonical sticky-wage model, and differs from it by the limit term:

$$\begin{aligned}\log(1+\pi_t^w) = \kappa_w N_t\Bigg[&-\int\frac{U_h(c_{it},H_t)}{\chi_{it}^{\infty}}\,di\\ &-(1-\tau_t^w)\tilde{w}_t\int\left(\frac{\epsilon_w-1}{\epsilon_w}+1-\chi_{it}^{\infty}\right)x_{it}U_c(c_{it},H_t)\,di\Bigg]\\ &+\beta\log(1+\pi_{t+1}^w).\end{aligned}$$

Thus, the NKWPC in the sticky-wage HANK with complementarity model differs from the canonical sticky-wage HANK by a term that reflects the effect reference hours have on labor earnings in the limit. Because the limit averages to one, that term is a covariance,

$$\int\left(1-\chi_{it}^{\infty}\right)x_{it}U_c(c_{it},H_t)\,di = -\operatorname{Cov}_x\left(\chi_{it}^{\infty},U_c\right).$$

The two wage Phillips curves coincide if and only if this covariance is zero. The covariance term is the limit of the multiplier on the reference hours constraint. Substituting $\Phi_{it} = 1 + (\chi_{it} - 1)/\rho$ into the expression for λ_t^H gives

$$\lambda_t^H = -(1 - \tau_t^w)\tilde{w}_t \frac{1-\rho}{\rho} \operatorname{Cov}_x (U_c, \chi) \longrightarrow (1 - \tau_t^w)\tilde{w}_t \operatorname{Cov}_x (U_c, \chi^\infty),$$

since $(1 - \rho)/\rho$ converges to minus one. The term by which the two wage Phillips curves differ is therefore the limit shadow price of the reference hours constraint. The shadow price does not vanish in the limit because even though dispersion in hours declines, its cost increases.

E.5. Fiscal Multipliers in the Sticky-Price and Sticky-Wage Models

When hours worked are complementary enough to match the MPE, whether prices or wages are sticky has a limited effect on fiscal multipliers. As Table E.1 shows, both the sticky-wage and sticky-price models generate moderate fiscal multipliers under the constant real rate rule and the three-year nominal interest rate peg.

Table E.1: Present-value multipliers under different monetary and fiscal regimes.

	Monetary regime			Fiscal regime		
	Taylor rule	Constant-r	3-year peg	$\rho_B = 0$	$\rho_B = 0.9$	$\rho_B = 0.95$
	Sticky prices with $\sigma_h \to \infty$					
Separable	0.77	1.29	73.92	1.29	1.29	1.29
$\alpha = 0.5$	1.14	2.49	3.49	2.49	2.49	2.49
GHH	2.39	5.50	6.00	5.50	5.50	5.50
	Sticky prices with separable preferences					
$\rho = -7.0$	0.81	1.24	1.80	1.24	1.24	1.24
$\rho = -1.962$	0.80	1.28	2.40	1.28	1.28	1.28
$\rho = -0.46$	0.77	1.24	2.53	1.24	1.24	1.24
$\rho = 1.0$	0.77	1.29	73.92	1.29	1.29	1.29
	Sticky wages with $\sigma_h \to \infty$					
Separable	0.89	0.91	0.94	0.75	0.91	1.15
$\alpha = 0.5$	0.89	0.99	1.12	0.81	0.99	1.27
GHH	1.15	1.34	1.57	1.13	1.34	1.68
	Sticky wages with separable preferences					
Uniform hours	0.93	1.18	1.46	0.94	1.18	1.59
$\rho = -7.0$	0.95	1.10	1.37	0.88	1.10	1.46
$\rho = -1.962$	0.95	1.04	1.25	0.84	1.04	1.37
$\rho = -0.46$	0.94	1.00	1.14	0.81	1.00	1.30
$\rho = 1.0$	0.89	0.91	0.94	0.75	0.91	1.15

Note: Different values of parameter α correspond to different degrees of complementarity between consumption and hours in preferences, with $\alpha = 0.0$ for separable preferences and $\alpha = 1.0$ for GHH preferences. Different values of ρ correspond to different degrees of complementarity between hours worked in production: $\rho = 1.0$ corresponds to standard linear aggregation, where hours are perfect substitutes, and values below zero indicate that hours worked are complements in labor aggregation.